OPTIMAL ESTIMATION OF ROUGHNESS IN OPEN-CHANNEL FLOWS

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ABSTRACT: The inverse problem of estimating the open-channel flow roughness is solved using an embedded optimization model. Measurement data for flow depths and discharges at several locations and times are used as inputs to the optimization model. The nonlinear optimization model embeds the finite-difference approximations of the governing equations for unsteady flow in an open channel as equality constraints. The Sequential Quadratic Programming Algorithm is used to solve the optimization model. The performance of the proposed parameter estimation model is evaluated for different scenarios of data availability and noise in flow measurement data. Solution results for illustrative problems indicate the potential applicability of the proposed model.

INTRODUCTION

An accurate estimation of Manning’s roughness coefficient is of primary importance in any study involving open-channel flow. However, the estimation of this coefficient for a natural channel is not a trivial task as it depends on several factors, including surface roughness characteristics, vegetation, channel irregularity, etc. Many empirical procedures (Urquhart 1975) have been suggested in the past for estimating the value of Manning’s coefficient.

It is possible to estimate Manning’s $n$ from field measurements of discharge and depth using optimization techniques. Parameter estimation techniques using unsteady flow data are more popular, because a large amount of data can be obtained easily from a limited number of gauging stations. Significant work has been reported in the area of ground water relating to parameter estimation (Willis and Yeh 1987). The literature relating to identification and estimation of parameters for unsteady open-channel flow is sparse. Becker and Yeh (1972, 1973) proposed a methodology for parameter estimation in unsteady open channel flow using the influence coefficient approach by minimizing the sum of squares of differences between observed data and numerically simulated values. Fread and Smith (1978) used a modified Newton-Raphson search technique for estimating the parameter $n$ as a function of stage and discharge. They minimized the absolute value of the sum of the differences between observed and computed stages and discharges. Fread and Smith (1978) applied the above algorithm sequentially to a multiple reach river system by specifying the computed discharge at the downstream end of the $m$th reach as the upstream boundary condition for ($m + 1$)th reach. The model proposed by Fread and Smith is not general in the sense that it can be applied to only dendritic river systems. Also, any errors of estimation in the upstream reaches, particularly due to observational errors, may significantly affect estimated values in the downstream reaches, because a sequential algorithm is adopted. Recently, Wasantha Lal (1995) used a singular value decomposition method by formulating the calibration problem as a generalized linear inverse problem. The singular value decomposition technique is applicable to underdetermined and overdetermined problems as well. However, this method also uses the influence coefficient approach for predicting the values of parameters during successive iterations. In all the previous works, it is necessary to solve the governing equations in a separate solver outside the optimization model. This may necessitate running the solver a large number of times iteratively, in order to determine the objective function value and the influence coefficient matrix. Also, convergence to an optimum solution may prove to be difficult if a large number of parameters are to be estimated.

In this study, a new approach is proposed to estimate roughness coefficients in a channel system by directly embedding the finite-difference approximations of the governing equations for flow into a nonlinear optimization model as equality constraints. The SQP algorithm (Powell 1974) is used to minimize the nonlinear objective function based on the least square error criterion. Potential applicability of the model is demonstrated by considering simulated observation data for hypothetical channels. Performance of the model is evaluated through a number of numerical experiments. Effect of observation noise on the performance of the model is also evaluated.

GOVERNING EQUATIONS

In this study, one-dimensional shallow water flow equations for a prismatic, wide rectangular channel characterized by a single weighted roughness coefficient at each cross section are used as governing equations. However, it is simple to extend the methodology to flows in irregular channels by appropriately modifying the governing equations. The governing equations are written as follows:

- Continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (1)$$

- Momentum equation

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} (q^2h + gh^2) = gh(x_0 - x_s) \quad (2)$$

where $h(x, t) = \text{flow depth (m)}$; $q(x, t) = \text{discharge per unit width of channel (m}^3\text{/s})$; $x_0 = \text{bottom slope of channel}$; $x_s = \text{friction slope}$; $x = \text{distance along flow direction (m)}$; and $t = \text{time (s)}$. The friction slope, $x_s$, is given by the Manning’s equation

$$x_s = \frac{n^2q^2}{h^{5/3}} \quad (3)$$

where $n = \text{Manning roughness coefficient}$.

FORMULATION OF PROBLEM

The inverse problem is formulated as a nonlinear optimization model with roughness coefficient ($n$) as an unknown parameter. The model is evaluated for different scenarios of data availability and noise in flow measurement data.
variable. The objective function is the sum of squares of the differences between the estimated and measured values of flow depth and/or discharge. Finite-difference analogs of the governing equations (1) and (2) and specified boundary conditions at the upstream and downstream ends constitute the equality constraints of the optimization model. Formulation of the inverse problem for a single reach channel can be summarized as follows.

**Objective Function**

Minimize

\[
\sum [(x(o) - x(m))/x(m)]^2
\]

where \(x(o)\) = simulated depth or discharge values at several observation stations; and \(x(m)\) = observed depth or discharge values at the corresponding points.

**Linear Constraints**

Finite difference analog of the continuity equation using the Preissmann scheme gives the linear constraints in the optimization model:

\[
\frac{(h_{i+1}^{k+1} + h_i^{k+1}) - (h_i^{k+1} + h_i^k)}{2\Delta t} + \frac{\theta (q_{i+1}^{k+1} - q_{i}^{k+1})}{\Delta x} + (1 - \theta) \frac{(q_{i+1}^{k+1} - q_i^k)}{\Delta x} = 0
\]

for \(i = 1\) to \(N-1\); \(k = 1\) to \(p\), where \(N =\) total number of grid points, \(p =\) total number of time steps in the computational domain, and \(\theta =\) weighting parameter in the Preissmann scheme. Usually a value of \(\theta\) between 0.5 and 1.0 is chosen to ensure numerical stability. In our computations, a value of 0.67 is found suitable for single channels, while a higher value of 0.8 is better suited for multiple channel system. The specified upstream boundary condition is also included as a linear constraint. This is given as

\[
q_{1}^{k} = f(t)
\]

where \(f(t)\) = specified inflow hydrograph.

**Nonlinear Constraints**

A finite-difference analog of the momentum equation using the Preissmann scheme forms the set of nonlinear constraints:

\[
\frac{(q_{i+1}^{k+1} + q_i^{k+1}) - (q_i^{k+1} + q_i^k)}{2\Delta t} + (\theta/\Delta x) \left( \frac{(q_{i+1}^{k+1})^2}{h_{i+1}^{k+1}} - \frac{(q_i^{k+1})^2}{h_i^{k+1}} \right) + \left( (1 - \theta)/\Delta x \right) \left( \frac{(q_{i+1}^{k+1})^2}{h_{i+1}^{k+1}} - \frac{(q_i^k)^2}{h_i^k} \right) - \left( h_i^{k+1} \right)^2 + \left( g(1 - \theta/\Delta t)(h_i^{k+1} - h_i^k) \right) + \left( g(2\Delta t)(h_i^{k+1}) \right) + \left( g(2\Delta t)(h_i^{k+1}) \right) - \left( g(\Delta t/2)/(h_i^{k+1} + h_i^k) \right) + \left( 1 - \theta \right)(h_i^{k+1} + h_i^k) + \left( g(\Delta t/2)/(h_i^{k+1} + h_i^k) \right) = 0
\]

for \(i = 1\) to \(N-1\); \(k = 1\) to \(p\). A rating curve, if available, can be specified as the boundary condition at the downstream end. In all the studies reported here, a uniform flow condition is assumed as the downstream boundary condition. This is given as

\[
q_{N}^{k+1} = \frac{(h_{N}^{k+1})^{1/3}\sqrt{n}}{n}
\]

Eq. (8) is also included in the set of nonlinear constraints. Formulation of the parameter estimation model for a channel system is very similar to the above described model for a single reach channel, with the compatibility conditions at junctions forming additional equality constraints. Mass balance and energy balance conditions at the junctions constitute these compatibility conditions. The nonlinear optimization problem described in this section is solved using the Sequential Quadratic Programming (SQP) algorithm (Powell 1974).

**PERFORMANCE EVALUATION**

Performance of the proposed optimization model for the estimation of roughness coefficient is evaluated using illustrative example problems for hypothetical open-channel systems. These example problems include flow in a single channel with a single value of \(n\) and a simple dendritic system of three channels with multiple roughness values corresponding to different reaches. The observation data for these cases are simulated by solving governing equations (1) and (2) for assumed true values of \(n\) using the Preissmann scheme. These simulated observation data for discharge and flow depth are then used in the optimization model to estimate the roughness coefficients. Identical initial and boundary conditions are applied while obtaining the simulated observation data and while solving the optimization model. The advantage of using simulated observation data is that it provides a means to delineate the actual deviation between the true values and the estimated values of \(n\). This approach enables us to evaluate the performance of the methodology also for the idealized situation when no measurement noise is present.

In order to evaluate the performance of the proposed methodology, several scenarios of observation data availability are considered. The first eight scenarios assume that the observed data are available for both \(q\) and \(h\), at each time step, and at each spatial grid point of the parameter estimation model. In the other scenarios, the performance of the methodology is also evaluated for situations where observed data are available at only few time steps and spatial grid points of the parameter estimation model. These scenarios and corresponding evaluation results are presented in the following sections.

**Single Channel Problem**

A wide rectangular channel with a constant bed slope of 0.0004 is considered in this case. The length of the channel is 40,000 m. A uniform flow depth, \(h = 2.5\) m, and a corresponding uniform flow discharge for \(n = 0.023\) are specified as the initial steady state conditions. A Log-Pearson Type III hydrograph is chosen arbitrarily as the upstream boundary condition. In this hydrograph, \(q_1 = \) initial unit discharge = 3.32 m\(^3\)/s; \(q_p = \) peak unit discharge = 6.0 m\(^3\)/s; \(t_s = \) time to peak = 2 h; \(t_c = \) time to centroid of hydrograph = 2.5 h; and \(t_r = \) time base of hydrograph = 6 h. Variation of \(q\) with time for this hydrograph is given by the following equation:

\[
q_i = q_o + (q_s - q_o) e^{-(t/t_o)\alpha} - (t/t_o)\alpha
\]

Flow measurement data for this case are simulated by numerically solving the governing equations (1) and (2) using the Preissmann scheme. The measurement data for this case are obtained at half-hour intervals (\(\Delta t = 0.5\) h) at measurement locations one thousand meters apart (\(\Delta x = 1,000\) m). These simulated noise-free observation data for \(q\) and \(h\) are used in the optimization model to estimate the roughness coefficient. A larger computational grid size is considered in the optimization model for reducing the computational costs even though...
measurement data are available at smaller grid sizes. Manning’s $n$ is estimated for different scenarios, as presented in Table 1, in order to study the grid size effect on the parameter estimation. The initial estimate of $n$, which is required to start the optimization model, is specified to be 0.01 in all the runs. Table 1 data suggests that the optimization model results in a variation in $n$ of 0.0001 for a variation of 0.5 h for $\Delta t$, and from 2,000 to 8,000 m in $\Delta x$. The maximum difference between the estimated $n$ value and the true value is only 0.0002. An average of 10 iterations of the optimization model are required to arrive at these optimal $n$ values. Estimated and observed stage hydrographs at $x = 20,000$ m for Scenario 1 in Table 1 are almost identical, as shown in Fig. 1. The optimization model performs satisfactorily even when the initial estimate is far from the true value (Ramesh 1997).

### TABLE 1. Gridsize Effect on Parameter Estimation

<table>
<thead>
<tr>
<th>Scenario number (1)</th>
<th>$\Delta x$ (m)</th>
<th>$\Delta t$ (h)</th>
<th>Estimated values of $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2,000</td>
<td>1.0</td>
<td>0.0229</td>
</tr>
<tr>
<td>2</td>
<td>4,000</td>
<td>1.0</td>
<td>0.0229</td>
</tr>
<tr>
<td>3</td>
<td>8,000</td>
<td>1.0</td>
<td>0.0299</td>
</tr>
<tr>
<td>4</td>
<td>4,000</td>
<td>0.5</td>
<td>0.0228</td>
</tr>
<tr>
<td>5</td>
<td>8,000</td>
<td>0.5</td>
<td>0.0228</td>
</tr>
</tbody>
</table>

![FIG. 1. Estimated and Observed Stage Hydrographs at X = 20 km (Scenario 1)](image)

Multiple Reach Channel System

A dendritic system of three wide rectangular channels, as shown in Fig. 2, is considered in this case. Each channel is divided into two reaches of equal length, with each reach having a different roughness coefficient. All three channels are of the same length = 12,000 m, with an identical bed slope, $s_0 = 0.0004$. The flow measurement data for this system are simulated by using the hydrograph of (9) as the upstream boundary condition for channels 1 and 2. The true values of $n$ used in these simulations are 0.018, 0.023, 0.028, 0.025, 0.031, and 0.035 for reaches 1, 2, 3, 4, 5, and 6, respectively. These flow measurement data are simulated using $\Delta t = 1,000$ m and $\Delta t = 0.5$ h and $h = 0.8$ in the simulator. The initial steady state conditions for this system are obtained by solving the gradually varied flow equation corresponding to unit discharges of $3.53 \text{ m}^3/\text{s}$, $2.27 \text{ m}^3/\text{s}$, and $5.80 \text{ m}^3/\text{s}$ in channels 1, 2, and 3, respectively. The initial downstream depth in channel 3 in these computations is 4.02 m. The simulated flow measurement data are used in the objective function of the optimization model to estimate the $n$ values for all six reaches. The initial estimates for all six $n$ values are specified to be 0.022.

Table 2 shows the effect of the grid size on the performance of the optimization model. It can be observed from Table 2 that the optimization model performs satisfactorily in this multiple channel system case. For example, the maximum error in the estimation is only 0.0013 (5.7%) for Scenario 6 ($\Delta t = 2,000$ m and $\Delta t = 1$ h). As may be expected, errors in the estimated values of $n$ increase as $\Delta t$ and $\Delta x$ are increased. Identical results are obtained when the initial estimates for $n$ are taken equal to 0.01, indicating the robustness of the proposed model. Estimated and observed stage hydrographs 6,000 m downstream of the junction point along channel 3 for Scenario 6 are compared in Fig. 3. Satisfactory performance of the optimization model can be observed again from this figure. The maximum difference between the estimated and observed flow depths is only 0.11 m.

In many cases, observation data may be available either for flow depths or discharges. Also, data may be available only at a few selected stations and a few selected times. However, it may be required to use a smaller $\Delta t$ and $\Delta x$ in the optimization model while estimating the parameters in order to reduce discretization errors. Performance of the optimization model under such conditions is studied by considering several scenarios, as presented in Table 3. $\Delta x$ and $\Delta t$ in all these studies are 2,000 m and 1 h, respectively. The initial estimates for the flow variables are taken as the spatial mean of the available data at that time level. Initial estimates for the $n$ values are taken as 0.022. Results presented in Table 3 indicate that the proposed model performed satisfactorily (maximum error = 13.2%) even when the availability of the measurement data was sparse (Scenarios 9–12). However, there was a significant deterioration in the performance of the model when the flow depth measurements were not available (Scenario 13). Results were also not satisfactory in the case of Scenario 14, where the number of observation stations was less than the number of parameters to be estimated.

In many cases, flow measurement data from field conditions

### Multiple Reach Problem

<table>
<thead>
<tr>
<th>Scenario number (1)</th>
<th>$\Delta x$ (m)</th>
<th>$\Delta t$ (h)</th>
<th>% Error in Estimated Values of $n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>2,000</td>
<td>1.0</td>
<td>Reach 1: 0.5, Reach 2: 5.7, Reach 3: 2.5, Reach 4: 1.2, Reach 5: 1.3, Reach 6: 4.0</td>
</tr>
<tr>
<td>7</td>
<td>6,000</td>
<td>1.0</td>
<td>Reach 1: 1.7, Reach 2: 10.9, Reach 3: 0.4, Reach 4: 5.6, Reach 5: 1.3, Reach 6: 3.4</td>
</tr>
<tr>
<td>78</td>
<td>2,000</td>
<td>2.0</td>
<td>Reach 1: 0.0, Reach 2: 0.4, Reach 3: 2.5, Reach 4: 10.4, Reach 5: 2.6, Reach 6: 6.6</td>
</tr>
</tbody>
</table>
may contain observation noise. Performance of the proposed parameter estimation model in the presence of observation noise is evaluated here. In the illustrative case considered here, all the input data except the flow measurement data are identical to those for Scenario 6. The simulated noise-free measurement data \( h(x, t), q(x, t) \) for Scenario 6 are modified by adding a random error term as follows:

\[
H'(x, t) = h(x, t) + \varepsilon(x, t); \quad q'(x, t) = q(x, t) + \varepsilon(x, t)
\]  

(10)

where the prime indicates noise measurement data input into the optimization model; and \( \varepsilon(\cdot) = \) a random error term sampled from a uniform distribution of zero mean, and the upper and lower limits of \( \pm \alpha h(x, t) \) [or \( q(x, t) \), as the case may be] and \( -\alpha h(x, t) \), respectively. In this evaluation, five sets of noisy flow measurement data are generated using this procedure. These data sets are then incorporated into the objective function of the parameter estimation model to estimate the roughness values. Average \( n \) values obtained as solutions for the five sets of noisy data with \( \alpha = 0.1 \) and \( 0.2 \) are close to those for \( \alpha = 0.0 \), representing the error free measurement, with the maximum difference being only 0.0016. Table 4 indicates the potential applicability of the proposed model for real-life situations where the measurement data are erroneous.

![Observed vs. Estimated Stage Hydrographs](image)

**FIG. 3.** Estimated and Observed Stage Hydrographs 6,000 m Downstream of Junction Point in Channel 3 (Scenario 6)

### TABLE 3. Performance Evaluation of Model for Multiple Channel Problem

<table>
<thead>
<tr>
<th>Scenario number (1)</th>
<th>Availability of observed data (2)</th>
<th>Estimated ( n ) values (3)</th>
<th>Maximum % error in ( n ) values (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>(I) both ( q ) and ( h )</td>
<td>0.0178, 0.0206</td>
<td>10.4</td>
</tr>
<tr>
<td></td>
<td>(II) ( t = 1, 3, ) &amp; ( 5 ) h</td>
<td>0.0206, 0.025</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(III) all spatial grid points</td>
<td>0.0299, 0.0359</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>(I) both ( q ) and ( h )</td>
<td>0.0182, 0.021</td>
<td>8.7</td>
</tr>
<tr>
<td></td>
<td>(II) ( t = 1 ) &amp; ( 4 ) h</td>
<td>0.0293, 0.0241</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(III) all spatial grid points</td>
<td>0.0307, 0.036</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>(I) both ( q ) and ( h )</td>
<td>0.018, 0.0206</td>
<td>13.2</td>
</tr>
<tr>
<td></td>
<td>(II) all times</td>
<td>0.0291, 0.0217</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(III) only the two end points of</td>
<td>0.0309, 0.0365</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a reach</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>(I) only ( h )</td>
<td>0.0183, 0.0236</td>
<td>9.7</td>
</tr>
<tr>
<td></td>
<td>(II) all times</td>
<td>0.0278, 0.0256</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>(I) only ( q )</td>
<td>0.0203, 0.0191</td>
<td>31.1</td>
</tr>
<tr>
<td></td>
<td>(II) all times</td>
<td>0.0367, 0.0238</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(III) all spatial grid points</td>
<td>0.0211, 0.0363</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>(I) both ( q ) and ( h )</td>
<td>0.0191, 0.0235</td>
<td>22.0</td>
</tr>
<tr>
<td></td>
<td>(II) all times</td>
<td>0.0276, 0.0197</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(III) only for midpoints of</td>
<td>0.0323, 0.0429</td>
<td></td>
</tr>
<tr>
<td></td>
<td>reaches 1, 3, &amp; 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### SUMMARY AND CONCLUSIONS

In this study, an embedded optimization model is proposed for estimating the Manning’s roughness coefficients from unsteady flow measurement data in open-channel systems. Finite-difference approximations of the governing flow equations are embedded as equality constraints in a nonlinear optimization model. Performance of the proposed model is evaluated for different scenarios of data availability and observation noise in the flow measurement data. The evaluation herein is limited to subcritical flow conditions. It is found that the model performs satisfactorily in all the cases except: (1) when only discharge measurements are available; and (2) when the number of observation stations is less than the number of parameters to be estimated. Results of these evaluations show the potential applicability of the proposed approach for estimating roughness in open channels. In our performance evaluation, the writers have used wide rectangular cross sections characterized by the same weighted roughness coefficient for the channel and overbanks, because many large natural channels (particularly in India) can be approximated as such. However, the methodology itself is not limited to wide rectangular channels and requires only slight modifications in the governing equations used. Further testing of the methodology using field data is necessary to establish its practical utility for natural channels.

### APPENDIX I. REFERENCES


### APPENDIX II. NOTATION

The following symbols are used in this paper:

- \( g \) = acceleration due to gravity (m/s²);
- \( h \) = flow depth (m);
- \( j \) = grid point in space;
- \( k \) = time level;
- \( N \) = total number of nodes;
\( n \) = Manning’s coefficient;  
\( p \) = total number of time steps;  
\( q \) = discharge per unit width of channel (m\(^2\)/s);  
\( S_0 \) = bottom slope of channel;  
\( s_f \) = friction slope;  
\( t \) = time (s);  
\( x \) = longitudinal distance (m);  
\( x(m) \) = observed value of depth or discharge;  
\( x(o) \) = simulated value of depth or discharge;  
\( \Delta x \) = grid spacing;  
\( \alpha \) = induced error percent;  
\( \Delta t \) = computational time step;  
\( \varepsilon \) = random error sampled from uniform distribution; and  
\( \theta \) = weighting coefficient in Preismann scheme.