TWO-DIMENSIONAL ANALYSIS OF DAM-BREAK FLOW IN VERTICAL PLANE

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ABSTRACT: This work presents numerical computations for the analysis of Dam-Break Flow using two-dimensional flow equations in a vertical plane. The numerical model uses the general approach of the simplified marker and cell method combined with the volume of fluid approach for the surface tracking. The time evolution of flow depth at the dam site and the evolution of the pressure distribution are investigated for both wet and dry bed conditions. The effect of the initially nonhydrostatic state on the long term surface profile and wave velocity are studied. These long term effects are found to be marginal in the case of wet-bed conditions, but are significant in dry-bed conditions. The dry-bed tip velocity immediately after the dam break, computed numerically, compares well with analytical results published previously. The time taken to obtain a constant flow depth at the dam site increases with decreasing initial depth ratio. The numerical result for this time elapsed for dry-bed conditions is close to the experimentally obtained value.

INTRODUCTION

The analysis of Dam-Break Flow (DBF) is a part of dam design and safety analysis. The importance of DBF analysis cannot be overemphasized, as dams are potential sources of hazard to life and property. DBF analysis is also crucial for flood plain management and before locating strategic or dangerous infrastructures in river valleys (Almeida and Franco 1994).

Studies to understand the basic mechanics of DBF date back to the earliest attempt by Ritter in 1892. Ritter (1892) derived an analytical solution for the hydrodynamic problem of instantaneous dam break in a frictionless and horizontal channel of rectangular shape. Later, Dressler (1952) and Whitham (1955) included the effect of bed resistance in the analysis of DBF and derived analytical expressions for the velocity and height of the wave front. Stoker (1957) extended the Ritter solution to the case of wet-bed conditions on the downstream side. He derived analytical expressions for the surface profile in terms of the initial depths, upstream and downstream of the dam. Ritter’s and Stoker’s solutions assume that the reservoir is infinite. On the other hand, the analytical equations derived by Hunt (1982, 1987) consider finite length reservoirs. However, Hunt’s solution has the limitation that it is based on the postulation of a kinematic wave. All the aforementioned analytical solutions are based on the assumption that the pressure distribution is hydrostatic and the vertical profile of velocity is uniform.

There has been a concern in the literature regarding the assumption of a hydrostatic pressure distribution in analyzing the mechanics of DBF. It has been well established that the pressure distribution is nonhydrostatic immediately after the dam failure (Pohle 1952; Strelkoff 1986). Dressler (1954) experimentally showed that the depth at the dam site does not attain a constant value instantaneously after the dam failure, as predicted by Ritter using the hydrostatic assumption. It takes approximately nine nondimensional time units to reach the constant Ritter value. Korosin (1983) has demonstrated that this nonhydrostatic pressure distribution reduces the wave speed by about 30%. All the above studies are for dry-bed, downstream conditions. However, there have been no comparable studies for wet-bed conditions. Even in the works cited above there are considerable gaps, as no study of the complete evolution of the pressure field after the dam break, and the short-term and the long-term consequences thereof, has been done. This work is an attempt to do a detailed numerical study of DBF for both dry-bed and wet-bed conditions using the two-dimensional Euler equations which allow the computation of the complete velocity and the pressure fields. No attempt is made to include the effects of turbulence in the simulation. Numerical simulations of DBF using two-dimensional equations have been reported earlier by various researchers (Harlow and Welch 1965; Amsden and Harlow 1970; Hirt and Nichols 1981; Chen et al. 1993; Nakayama and Mori 1996), but only for dry-bed downstream conditions. Furthermore, DBF was used only as a test case in all these studies and no detailed analysis was presented. Strelkoff (1986) reports an exploratory numerical study on DBF, also for dry-bed conditions, performed by Nichols using the Hirt and Nichols (1981) method.

In this paper, we first present a brief description of the numerical algorithm for simulating free surface flows, and the verification of the algorithm using the analytical solution for a plane progressive wave. The numerical algorithm used for this study is a composite one developed using the GENSMAC Navier-Stokes Solver (Tome and McKee 1994) combined with the Y-VOF (VOF = volume of fluid) surface tracking technique (Rudman 1997). The surface tracking of a dam break wave poses a formidable challenge to the tracking algorithm, and many of the currently used surface tracking techniques were found to be insufficiently accurate for our purpose. The VOF (Hirt and Nichols 1981) and its improved version Y-VOF (Rudman 1997) were judged to be the best of the current surface tracking methods for finite-difference schemes (Rudman 1997) and are hence used in this study. The GENSMAC solver for time stepping the velocity and the pressure fields is used here for its high computational efficiency. Mechanics of DBF for both wet- and dry-bed conditions is analyzed based on numerical simulations. Emphasis is laid on the time evolution of the flow depth at the dam site and the evolution of the pressure distribution toward a hydrostatic state. The effect of the initially nonhydrostatic state on the long term surface profile and the wave velocity are also studied. The numerical results for the wave speed and the time period of the initial nonhydrostatic state are also compared with the analytical and experimental data available in the literature.

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GOVERNING EQUATIONS

The equations governing the two-dimensional, time-dependent, incompressible inviscid flows are the Euler equations. These equations, in nondimensional form, are

continuity equation:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$  \hspace{1cm} (1)

momentum equation in x direction:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^* u^*}{\partial y^*} - \frac{\partial p^*}{\partial x^*} + \frac{g_y^*}{G_n^*} = 0$$  \hspace{1cm} (2)

momentum equation in y direction:

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial u^* v^*}{\partial x^*} - \frac{\partial p^*}{\partial y^*} + \frac{g_x^*}{G_n^*} = 0$$  \hspace{1cm} (3)

in which $G_n^* = v^2 / L g$; $u^*$ = nondimensional velocity in x direction = $u V / L$; $v^*$ = nondimensional velocity in y direction = $v V / L$; $p^*$ = nondimensional pressure = $p / \rho V^2$; $g_x^*$ = nondimensional gravity component in x direction = $g / g$; $g_y^*$ = nondimensional gravity component in y direction = $y L / g$; $x^*$ = $x / L$; $y^*$ = $y L$; $t^*$ = $t T$; $L$ = length scale; $V$ = velocity scale; and $T$ = time scale = $L / V$. The coordinate axes x and y are in the horizontal and the vertical directions, respectively. For simplicity, the superscript (*) mark is omitted in the rest of the text.

In this study, we do not make any attempt to include the effects of turbulence, although dam-break flows are well known to be turbulent. It is expected that the turbulence would cause diffusion effects, which are minor when compared with the highly convective nature of the flow. The solution of this two-dimensional problem with a strong shock is quite challenging in itself; the incorporation of turbulence would add another level of difficulty. It should also be borne in mind that the model cannot be used for determining the roughness effects on flow characteristics. The purpose here is only to study the effect of nonhydrostatic pressure distribution.

NUMERICAL METHOD

The basic principle of the method presented here follows the simplified marker and cell (SMAC) (Amsden and Harlow 1970) method of splitting the governing equations into two parts: (1) to predict the velocity field using explicit time stepping; and (2) to compute the pressure-correction field, which then corrects the velocity to satisfy the zero divergence condition. However, like the GENSMAC algorithm (Tome and McKee 1994), it uses the Conjugate Gradient method for solving the pressure correction equation. It also uses the VOF technique of Youngs (Rudman 1997) for determining the free surface location.

The method uses a finite-difference approach to solve the equations on a staggered grid. A typical cell arrangement with the variables is shown in Fig. 1. In Fig. 1(a), ‘f’ indicates a full cell, ‘s’ indicates a surface cell, and ‘e’ indicates an empty cell. The velocities are specified at the cell faces while the pressure is specified at the cell center [Fig. 1(b)].

Given that the velocity field and surface profile are known at any time level $t(t_{ini}$ initially), the velocity and pressure fields and the free surface are to be determined for the new time level, $t + \Delta t$. The velocity terms are explicit, computed using known values, but the pressure term is implicit, based on the unknown pressure values at $t + \Delta t$. The following procedure is adopted for this purpose.

Step 1. We assume an arbitrary pressure field, $p_{ini}$, for the unknown time level, but ensure the correct specification of the pressure for the surface cells. (The pressure at the surface is assumed to be $p_{atmospheric}$.)

Step 2. Predictor Step: We calculate the predicted velocity field, $\tilde{u}_{i,j}$ and $\tilde{v}_{i,j}$, at time $t + \Delta t$ by explicitly solving the momentum equations, using the known velocities, $u_{i,j}$ and $v_{i,j}$, at $t$ and the assumed pressure field, $p_{i,j}$.

Step 3. Corrector Step: We solve a Poisson equation to find the pressure correction $\psi$ and to correct the predicted velocities to get the final velocities that satisfy the continuity equation. The correct pressure field at $t + \Delta t$, $p$, is obtained by adding the pressure correction, $\psi$, to the previously assumed pressure field, $\bar{p}$.

Step 4. Surface tracking is done with the new velocity field using the VOF approach.

Steps 1 and 2 are similar in spirit to the SMAC predictor step (Amsden and Harlow 1970), step 3 to the GENSMAC pressure correction procedure (Tome and McKee 1994), and step 4 to the VOF surface tracking method of Youngs (Rudman 1997).

The initial velocity field, $u_{i,j}$ and $v_{i,j}$, the initial pressure $p_{i,j}$, and the initial position of the free surface, $h$, are specified by the initial conditions.

Predictor Step

Unlike in the SMAC or GENSMAC method, upwinding is introduced in the present study to achieve a better numerical behavior. For example, the finite-difference approximations for the convective terms in the x-momentum equation, (2), are discretized as

$$\frac{\partial u}{\partial x} \leftarrow 0.5 \left[ u_{L} + u_{R} + \alpha \text{sgn}(u_{i,j}) (u_{L} - u_{R}) \right]$$  \hspace{1cm} (4)

$$\frac{\partial uv}{\partial y} \leftarrow 0.5 \left[ u_{L} v_{i} + u_{R} v_{i} + \alpha \text{sgn}(u_{i,j}) (u_{L} v_{i} - u_{R} v_{i}) \right]$$  \hspace{1cm} (5)
where

\[ uL = \frac{\tilde{u}_{i+1,j}^2 - \tilde{u}_{i,j}^2}{\Delta x} \]  

(6)

\[ uR = \frac{\tilde{u}_{i,j+1}^2 - \tilde{u}_{i,j}^2}{\Delta y} \]  

(7)

\[ uB = \frac{\tilde{u}_{i,j} - \bar{u}_{i,j}}{0.5*vB} \]  

(8)

\[ uT = \frac{uT*vi - \bar{u}_{i,j}}{0.5*vB} \]  

(9)

\[ uB = 0.5*(\tilde{u}_{i,j} + \tilde{u}_{i,j+1}) \]  

(10)

\[ uT = 0.5*(\tilde{u}_{i,j} + \tilde{u}_{i,j+1}) \]  

(11)

\[ vB = 0.5*(\tilde{v}_{i,j-1} + \tilde{v}_{i,j+1}) \]  

(12)

\[ vT = 0.5*(\tilde{v}_{i,j-1} + \tilde{v}_{i,j+1}) \]  

(13)

\[ vav = 0.5*(vT + vB) \]  

(14)

while the pressure and the transient terms are forward-differenced, i.e.,

\[ \frac{\partial p}{\partial x} \leftarrow \frac{\phi_{i+1,j} - \phi_{i,j}}{\Delta x} \]  

(15)

\[ \frac{\partial u}{\partial t} \leftarrow \frac{u_{i+1,j} - u_{i,j}}{\Delta t} \]  

(16)

In (4) and (5), \( \alpha = \) upwinding parameter (\( \alpha = 0 \) for pure centered difference and zero upwinding, and \( \alpha = 1 \) for complete upwinding); and \( \text{sgn}(\tilde{u}_{i,j}) = \) sign of velocity \( \tilde{u}_{i,j} \). Similar approximations are made for the terms in the y-momentum equation, (3).

To compute a predicted cell velocity, \( \tilde{u}_{i,j} \) or \( \tilde{v}_{i,j} \), we need the adjacent cell velocities at time level \( t \). In Figs. 2(a and b), the \( \square \) mark indicates the unknown velocity to be computed, and the \( \circ \) mark indicates the values at the known time level that are needed for this computation. It is obvious that \( \tilde{u}_{i,j} \) and \( \tilde{v}_{i,j} \) can be computed only for the internal cells, i.e., \( \tilde{u}_{i,j} \) can be computed if \( (i + 1, j) \)th cell is not empty (i.e., it is either a full cell or a surface cell) and \( \tilde{v}_{i,j} \) can be computed if \( (i, j + 1) \)th cell is not empty. The velocities for the boundary cells cannot be computed by the momentum equations and therefore are calculated by applying the boundary conditions.

Boundary conditions for velocities: On the inflow boundary and on the outflow boundary (which is taken far downstream), both \( \tilde{u}_{i,j} \) and \( \tilde{v}_{i,j} \) are prescribed. On the channel bed boundary, the appropriate boundary condition for the Euler equations, the free slip condition is applied. A reflection technique (using fictitious points) is used for implementing this boundary condition.

In a free surface flow, the choice of surface boundary conditions is of great importance. As in the SMAC method (Amsden and Harlow 1970), the tangential stress condition and the continuity equation are applied to obtain the surface cell velocities in this study.

**Corrector Step**

The predicted velocities, \( \tilde{u}_{i,j} \) and \( \tilde{v}_{i,j} \), are computed for all the cells using (2) and (3). However, this velocity field does not satisfy the continuity condition, because the velocities are computed using an arbitrarily assumed pressure field, \( \phi_{i,j} \), for \( t + \Delta t \). To enforce the continuity condition this predicted velocity field has to be corrected. The procedure used by Tome and McKee (1994) in their GENSMAC algorithm is followed in the present study. A pressure correction \( \psi \) is defined such that the correct velocities at \( t \) + \( \Delta t \), \( u_{i,j} \) and \( v_{i,j} \), are obtained from the predicted values, \( \tilde{u}_{i,j} \), \( \tilde{v}_{i,j} \), by

\[ u_{i,j} = \tilde{u}_{i,j} - \frac{\partial \phi}{\partial x} \]  

(17)

\[ v_{i,j} = \tilde{v}_{i,j} - \frac{\partial \psi}{\partial y} \]  

(18)

If \( u_{i,j} \) and \( v_{i,j} \) satisfy the continuity equation, then \( \psi \) has to be a solution of the Poisson equation.

\[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \]  

(19)

The right hand side of the Poisson equation (19) is evaluated using the predicted velocity field, and the equation is then solved for the pressure correction, \( \psi \). Like the pressure, \( p \), \( \psi \) is defined at the cell center. The correct pressure \( p_{i,j} \) at the time \( t + \Delta t \) is now given by

\[ p_{i,j} = \phi_{i,j} + \frac{\psi_{i,j}}{\Delta t} \]  

(20)

As in the GENSMAC algorithm (Tome and McKee 1994), the conjugate gradient method (Press et al. 1993) is used in this study for solving (19), discretized using center-differences. For inflow, outflow, and rigid bed boundaries, the normal gradient of \( \psi \) is zero, i.e., \( \partial \psi/\partial n = 0 \), as correct normal velocities are known at these boundaries. At the free surface, \( \psi = 0 \) because the correct pressure is prescribed there.

It is important to prescribe the correct pressure for a surface cell. In SMAC and GENSMAC it is obtained by applying the
normal stress condition. However, as in SOLA-VOF, an extrapolation procedure is used here to prescribe the pressure in the surface cells.

**Surface Tracking**

The position of the free surface for the new time level is to be computed using the corrected velocities $u_i$ and $v_j$, at $t + \Delta t$. For this purpose, Young's VOF approach (Rudman 1997), which is a modified version of the method first proposed by Hirt and Nichols (1981), is used. The equation governing the evolution of the parameter $F$, representing the fraction of fluid in the cell, is

$$\frac{\partial F}{\partial t} + \frac{\partial Fu}{\partial x} + \frac{\partial Fv}{\partial y} = 0 \quad (21)$$

Eq. (21) is not solved by the usual finite-difference approach, because the sharp definition of the free boundaries may be lost by numerical diffusion. Therefore, the fluxes are computed for all the cells using first-order upwinding. The outward fluxes of all surface cells are then refined using the orientation of the surface within the cell. Sixteen types of surface orientations are possible depending on the $F$ values of the cell and its neighbors, and separate procedures are used for each type (Rudman 1997). In this work, both SOLA-VOF (Hirt and Nichols 1981) and Y-VOF (Rudman 1997) were used, but almost identical results are obtained with both the methods, so only the results of Y-VOF are presented.

**Stability**

The time stepping follows an explicit procedure, and is therefore governed by a stability criterion. For this purpose, $\Delta x$ is calculated before the start of each computational cycle by considering two separate time steps prescribed by the velocity in $x$-direction, and the velocity in $y$-direction, respectively (Tome and McKee 1994):

$$\Delta t_s = \frac{\Delta x}{|u|_{\text{max}}} \quad (22)$$

$$\Delta t_s = \frac{\Delta y}{|v|_{\text{max}}} \quad (23)$$

The actual time step is chosen as the minimum of these:

$$\Delta t = \min[\Delta t_s, \Delta t_y] \quad (24)$$

**MODEL VALIDATION**

The numerical algorithm described in the previous section is validated by comparing the numerical results with the analytical solutions for the surface profile, the velocity field, and the pressure field, of a plane progressive wave. It is a good test problem as the pressure distribution in a plane progressive wave is nonhydrostatic and the velocity distribution at a section is nonuniform. The wavy surface also provides a check for the surface tracking algorithm.

The analytical solutions for a plane progressive wave (Fig. 3) used here are based on the assumption that the flow is potential; that the second order wave effects are negligible, i.e., $2\pi A^2/\lambda \ll 1.0$ ($A$ = amplitude, $\lambda$ = wave length); and that the deep water effects are present, but the flow depth is finite. These equations are given as (Liggett 1993):

$$\eta = A \sin(kx - \sigma t) \quad (25)$$

$$u = Ak \sqrt{\frac{\phi \rho}{2\pi}} \sin(kx - \sigma t) \quad (26)$$

where $A$ = amplitude; $\lambda$ = wave length; $d$ = equilibrium depth; $k$ = wave number $= 2\pi/\lambda$; $\sigma$ = frequency; and $\eta$ = free surface elevation above the equilibrium surface. Eqs. (25)–(27) are valid for $d/\lambda > 0.277$.

A plane progressive wave with $\lambda = 20$ m, $d = 18.0$ m, and $A = 0.25$ m is considered for study. In the numerical computation, the exact flow field at time $t = -\pi/2\sigma$ are prescribed as the initial conditions and the pressure field, the velocity field, and the free surface elevation after one time period, i.e., $t = 3\pi/2\sigma$, are determined. The computational domain is chosen equal to two wave lengths and therefore periodic boundary conditions are applied at the inlet ($x = 0$) and at the outlet ($x = 2\lambda$). The analytical equations for velocity are used to prescribe the bottom boundary condition at $y = -d$. Zero pressure is prescribed as the boundary condition on the surface. The numerical computations are performed with $\Delta x = 0.50$ m and $\Delta y = 0.25$ m. Grid convergent results are obtained with the above grid size. The numerical results are compared with the analytical solutions for (1) the surface profile; (2) the horizontal velocity distribution at $x = \lambda$; (3) the vertical velocity distribution at $x = 3\lambda/4$; and (4) the pressure distribution at $x = \lambda$ in Figs. 4(a–d), respectively. It can be observed from these figures that the numerical results for the velocity and the pressure distributions [Figs. 4(b–d)] match excellently with the corresponding analytical solutions. The hydrostatic pressure distribution is also shown in Fig. 4(d). It can be seen that the correct pressure distribution (as given by the analytical as well as the numerical solutions) is different from the hydrostatic distribution. The comparison between the computed and the analytical surface profiles [Fig. 4(a)] is also good. The maximum error occurs near the peaks and troughs and is at most equal to only 0.2%. These results indicate the satisfactory performance of the numerical algorithm. Though not presented here, almost identical numerical results are obtained using the VOF technique of Hirt and Nichols (1981) for the surface tracking. The matching results using two different techniques builds further confidence in our computational scheme.

**RESULTS AND DISCUSSION**

The numerical algorithm presented in the previous sections is now used for simulating and analyzing the DBF caused by the instantaneous rupture of the dam shown, schematically in Fig. 5. All the computations in this study are performed using the VOF technique of Hirt and Nichols (1981) for the surface tracking. The matching results using two different techniques builds further confidence in our computational scheme.
FIG. 4. Comparison of Analytical and Numerical Results for Plane Progressive Wave: (a) Surface Profile; (b) Horizontal Velocity, \( u \) at \( x = \lambda \); (c) Vertical Velocity, \( v \) at \( x = 3/4 \lambda \); (d) Pressure, \( p \) at \( x = \lambda \)

FIG. 5. Definition Sketch for Dam-Break Flow Problem

for four different values of \( h_d \) (\( h_d \) = the nondimensional depth downstream of the dam prior to the dam failure) equal to 0.0, 0.2, 0.5, and 0.7. The channel bed slope is equal to zero in all the computations. The computations are performed using \( \alpha = 1.0 \), \( \Delta x = 0.0667 \), and \( \Delta y = 0.05 \). This grid size gives grid convergent results. The initial conditions of zero velocity and known depths are kept unchanged at the inflow and the outflow boundaries as the boundary conditions during the transient computations. Note that the wave does not arrive at these points for the simulation time considered. All the computations are performed for a nondimensional time of 15.

Long-Term Effects

It is well known that the pressure distribution is non-hydrostatic immediately after the dam failure. However, shallow water wave models are commonly used for simulating the dam-break flows, thereby assuming that the initial non-hydrostatic state does not affect the long-term results. This assumption underlying the applicability of shallow water wave models has not been rigorously tested either. On the other hand, some researchers (e.g., Kosorin 1983) contend that the initial two-dimensional effects have a bearing on the long-term results. In this section, we compare the numerical results for the long-term free surface profiles and the front velocity with the analytical solutions obtained using the hydrostatic pressure assumption.

The free surface profile at \( t = 15 \) is shown in Fig. 6 for four
different depth ratios, \( r = h_d/h_u \). In this figure, \( x \) represents the nondimensional distance from the dam, and \( h \) the nondimensional flow depth. Fig. 6(a) shows the nondimensional surface profiles for the case of dry downstream bed obtained both numerically by this study and analytically by Ritter (1892). Figs. 6(b–d) present the nondimensional surface profiles obtained by our simulations, and analytically by Stoker (1957), for \( r = 0.2, 0.5, \) and \( 0.7 \), respectively. These figures show all the four zones of a DBF very clearly. It can be observed from Figs. 6(a–d) that, on the whole, the analytical solutions for wet-bed conditions based on the hydrostatic assumption match quite well with the numerical results obtained using the more sophisticated two-dimensional model. The analytical solutions for the height and location of the wave front [Figs. 6(b–d)] match closely with the numerical results, the maximum difference in the front location being only 2%. However, the location of the tip in the case of dry bed [Fig. 6(a)] is overpredicted by the analytical model, as compared with the numerical result. The difference between the two is as high as 20%. These results indicate that Kosorin’s (1983) concern regarding the applicability of shallow water wave models for dam-break waves is warranted in the case of dry beds.

It is important to mention here that the present numerical algorithm results in high-frequency oscillations near the steep wave front in the case of wet-bed conditions [Figs. 6(b–d)]. The origin of these oscillations can be only speculated upon at this point of time. They may be due entirely to numerical errors or, perhaps, to a combination of numerical errors and physical effects. Almeida and Franco (1994, pp. 355–357) report experimental studies that showed the existence of oscillations in case of dam-break flows with the depth ratio, \( r \geq 0.5 \). It is interesting to note that the solution of the one-dimensional Serre equations (which include the effect of non-hydrostatic pressure) also shows oscillations (Carmo and Santos 1993). In light of the above discussion, it may be improper to suppress these oscillations using smoothing techniques that don’t differentiate between numerical and physical oscillations. However, these oscillations can be suppressed, if desired, by introducing artificial viscosity into the numerical solution by adding viscous terms to the momentum equations. Fig. 7 shows the enlarged water surface profile at \( t = 15 \), for depth ratio, \( r = 0.5 \), obtained using different values of artificial viscosity coefficient, \( k \). It can be seen from this figure that the height and location of the front are not affected significantly, even if the oscillations are suppressed. However, in this study we do not draw any conclusions that could be affected by the oscillations. The primary variables, the front speed, the front height, and the pressure profiles away from the front were found to be unaffected by these oscillations.

The distance traveled by the front as a function of time is shown in Fig. 8 for dry and wet downstream bed conditions (\( r = 0.5 \)). In this figure, \( t \) represents the nondimensional time after the collapse of the dam, and \( x \), the nondimensional distance traveled by the wave-front. The slope of this curve gives the velocity of the tip (dry-bed case) or the velocity of the wave-front (wet-bed case). It can be clearly seen from Fig. 8(a) that the two-dimensional effects reduce the rate at which the tip advances on a dry bed. It is found from Fig. 8(a) that, on average, the tip advances with a nondimensional speed of 1.6 rather than a value of 2 as suggested by Ritter (1892), while the initial speed is approximately 1.3. This is consistent with the finding of Kosorin (1983), who predicted an initial
Short-Term Effects

Evolution of Flow Depth at Dam Site

Both Ritter’s (1892) and Stoker’s (1957) solutions, which use the hydrostatic assumption, predict that the flow depth at the dam site attains a constant value instantaneously upon the dam break. However, Dressler (1954) has shown experimentally that it takes nine nondimensional time units for the dam-site depth to attain this constant value. The variation with time, \( t \), of the flow depth at the dam site, \( h \), obtained using the numerical algorithm, is shown in Fig. 9. Results for all four

tip speed of 1.41. This result clearly indicates a significant long-term effect of nonhydrostatic pressure distribution, in the case of dry-bed conditions.

For the wet-bed case [Fig. 8(b)], the front velocity in the Stoker solution matches well with the long-term front velocity in the two-dimensional solution. Whatever the difference is between the two-dimensional and Stoker solutions, it is discernable only in the initial stages. However, this difference is only marginal. This again shows that the initial nonhydrostatic state does not affect the long-term results significantly in the case of wet-bed conditions.
FIG. 12. Evolution of Pressure Distribution at Dam Site: (a) $r = 0.0$; (b) $r = 0.5$
depth ratios are presented. The two-dimensional numerical results clearly indicate that it takes some time for the depth at the dam site to attain a constant value. This time increases as the depth ratio decreases. For dry-bed conditions, i.e., for \( r = 0 \), the time taken to attain the constant depth as obtained numerically is approximately equal to 8.0. This is close to the value of 9.0 determined experimentally by Dressler (1954).

**Evolution of Pressure Distribution**

Fig. 10 shows the pressure history on the floor at the dam site for the dry-bed case. In this figure, the nondimensional pressure is plotted as a function of the nondimensional time. As expected, the pressure becomes zero immediately after the dam break because of the existence of free surface conditions. It starts increasing as the water starts flowing in the downstream direction. However, the pressure is not equal to the hydrostatic pressure, due to the stream-line curvature. It eventually approaches the hydrostatic value as time progresses. Similar results were obtained for the case of wet-bed conditions. Fig. 11 shows the pressure history on the floor at a distance \( x = -5 \), for the wet-bed case with \( r = 0.5 \). The pressure is equal to the hydrostatic value until the negative wave in the reservoir reaches this point (at \( t = 2.0 \)). Then the pressure becomes less than the hydrostatic value, because of convex stream-line curvature. However, the pressure eventually approaches hydrostatic value again as the stream-line curvature reduces with time. In the case of dry bed (not shown here), the pressure is less than the hydrostatic value for longer periods of time. The evolution of nondimensional pressure distribution at the dam site for both the dry- and the wet-bed conditions (\( r = 0.5 \)) are presented in Fig. 12. The pressure distribution at nondimensional times \( t = 1, 5, \) and 10 are shown in this figure. It is clear from Fig. 12 that the pressure is nonhydrostatic immediately after the dam break. However, it is interesting to note that the evolution to the hydrostatic condition is quicker in the case of wet-bed conditions. This is consistent with the previous observation of early attainment of constant dam-site depth for wet-bed conditions.

**CONCLUSIONS**

In this paper, a detailed analysis of the DBF mechanics for both wet- and dry-bed conditions is presented. This analysis is based on the numerical results obtained using two-dimensional flow equations that allow the computation of the complete velocity and the pressure fields. The numerical algorithm used for this purpose is a composite one developed using the GENSMART Navier-Stokes solver (Tome and McKeen 1994) and Y-VOF technique (Rudman 1997) for surface tracking. Following are the conclusions drawn from the DBF study:

1. The nonhydrostatic pressure conditions that prevail immediately after the dam break do not have any significant bearing on the long-term results in the case of wet-bed conditions. Models based on the hydrostatic pressure assumption give results as good as those of the two-dimensional model, as far as the front height and the front location are concerned.
2. In the case of dry-bed conditions, models based on the hydrostatic pressure assumption overpredict the long-term rate of advance of the tip by as much as 25\%. The nondimensional speed of the tip obtained using the two-dimensional model is 1.6. The average value of the initial advance rate of the tip as predicted by the two-dimensional model is equal to 1.3, which is close to the value 1.41 suggested by Kosorin (1983).
3. The evolution of the pressure distribution toward the hydrostatic state is quicker in the case of wet-bed conditions than with dry-bed conditions. In the drawdown zone upstream of the dam location, the pressure distribution remains nonhydrostatic for a longer time after the dam break for the case of dry beds.
4. The flow depth at the dam site attains a constant value equal to that predicted by the Stoker solution (1957). However, it does not attain this value instantaneously with the dam break. The time taken to attain the constant value at the dam site increases with decreasing depth ratio, \( r \). In the case of dry beds (\( r = 0 \)), the present two-dimensional model predicts that it takes eight nondimensional time units for the dam-site depth to attain a constant value. This matches satisfactorily with the experimental value of nine measured by Dressler (1954).

**APPENDIX I. REFERENCES**


**APPENDIX II. NOTATION**

The following symbols are used in this paper:

\[ a = \text{amplitude of wave}; \]
\[ d = \text{height of equilibrium free surface from bottom}; \]
\( F \) = volume of fluid function for cell;  
\( k \) = wave number;  
\( K \) = viscosity coefficient for smoothing;  
\( h_i \) = initial flow-depth downstream of dam;  
\( h_{it} \) = flow-depth at any location \( i \);  
\( h_u \) = initial flow-depth upstream of dam;  
\( p \) = actual pressure field at time level \( t + \Delta t \);  
\( u, v \) = velocities at time level \( t + \Delta t \);  
\( \bar{u}, \bar{v} \) =predicted velocities at time level \( t + \Delta t \);  
\( \bar{u}, \bar{v} \) = velocities at time level \( t \);  
\( \phi \) = assumed pressure field at time level \( t + \Delta t \);  
\( \psi \) = pressure corrections;  
\( \eta \) = free surface elevation of plane progressive wave;  
\( \sigma \) = frequency of plane progressive wave; and  
* = nondimensional values.

**Subscripts**

\( i, j \) = cell \( i, j \); and  
\( s \) = variable value at surface.