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Conjunctive surface–subsurface modeling of overland flow

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In this paper, details of a conjunctive surface–subsurface numerical model for the simulation of overland flow are presented. In this model, the complete one-dimensional Saint-Venant equations for the surface flow are solved by a simple, explicit, essentially non-oscillating (ENO) scheme. The two-dimensional Richards equation in the mixed form for the subsurface flow is solved using an efficient strongly implicit finite-difference scheme. The explicit scheme for the surface flow component results in a simple method for connecting the surface and subsurface components. The model is verified using the experimental data and previous numerical results available in the literature. The proposed model is used to study the two-dimensionality effects due to non-homogeneous subsurface characteristics. Applicability of the model to handle complex subsurface conditions is demonstrated. © 1998 Elsevier Science Limited. All rights reserved

Key words: Overland flow, Saint-Venant equations, Richards equation, Conjunctive model.

1 INTRODUCTION

Overland flow is a dynamic part of a watershed response to the rainfall and has many engineering applications, such as (i) water budgeting, (ii) flood prediction, (iii) soil erosion studies, (iv) investigations into infiltration process, (v) transport of pollutant to streams, and (vi) design of runways and parking lots. Mathematical modeling of overland flow involves the solution of the governing equations for both the surface flow and the groundwater flow with seepage at the ground surface acting as the connecting link. The level of model sophistication depends on the assumptions made for simplifying the governing equations before they are solved. Many models consider only the surface flow system along with an empirical treatment of the infiltration term. On the other hand, some models consider both the surface and the subsurface flow systems, but use either a diffusion wave or a kinematic wave approximation for modeling overland flow.

Akan 3,4, Sunada and Hong 30, De Lima and Van der Molen 13, and Govindaraju et al. 17 developed overland flow models based on the kinematic wave approximation.

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It should be noted here that the kinematic wave approximation fails for highly subcritical flows on flat slopes and when the downstream boundary condition is an important factor 25. Diffusion wave models are applicable over a wider range of flow conditions and, therefore, may be used for such cases. Hromadka et al. 21 developed a two-dimensional diffusion wave model assuming a constant effective rainfall intensity. Govindaraju et al. 15 derived an approximate analytical solution to overland flow under a specified net lateral inflow using the diffusion wave approximation. They also provide the complete numerical solutions for the diffusion wave equation. Todini and Venutelli 32 also developed a similar two-dimensional diffusion wave model in which the governing equations were solved by the finite-difference as well as the finite-element methods.

Vieira 33 conducted a comprehensive study of various approximations usually applied to the Saint-Venant equations. It was suggested that the complete Saint-Venant equations should be used in cases where the kinematic wave number is less than 5. Although, the overland flow models based on the complete Saint-Venant equations are computationally very intensive, they are more versatile and can also act as standards of comparison for simplified models. Among the earliest full dynamic models are those developed
by Woolhiser and Liggett \cite{31} and by Lin et al. \cite{24}. Chow and Ben-Zvi \cite{12} and Kawahara and Yokoyama \cite{22} developed two-dimensional overland flow models for impervious surfaces based on the Lax–Wendroff method \cite{23} and a finite-element method, respectively. Zhang and Cundy \cite{37} developed a two-dimensional overland flow model for a temporally constant, but spatially varying infiltration. MacCormack finite-difference method was used to obtain the numerical solution. Tayfur et al. \cite{33} also developed a two-dimensional overland flow model based on an implicit method \cite{9} for solving the governing equations and applied the model to experimental hill slopes. It was reported that consideration of microtopography in fine detail could result in numerical instability and convergence problems.

All the overland flow models discussed so far describe the infiltration process in an empirical way. Smith and Woolhiser \cite{29} were probably the first to develop a conjunctive model for overland flow. They solved the one-dimensional Richards equation for unsaturated flow in the subsurface together with the one-dimensional kinematic wave equation for the surface flow. The Richards equation was solved by an implicit Crank–Nicolson finite-difference formulation. Akan and Yen \cite{5} developed a sophisticated conjunctive model in which they solved the complete Saint-Venant equations and the two-dimensional Richards equation. The Saint-Venant equations were solved by the four-point implicit method, while Richards equation was solved by the SLOR technique. In the very versatile, complete catchment model, SHE\cite{2}, the overland flow component is simulated using the diffusion wave approximation. The subsurface flow in the unsaturated zone is represented by the one-dimensional Richards equation. The surface flow equations are solved by the explicit procedure developed by Preissmann and Zaoui \cite{28}, while the Richards equation is solved by the implicit finite-difference scheme. Govindaraju and Kavvas \cite{16}, in a detailed study of hillslope hydrology through a stream flow—overland flow—subsurface flow model, used the diffusion wave approximation for the surface flow component. The surface flow equation was solved by the centred implicit scheme, while the two-dimensional Richards equation was solved by the SLOR technique.

In this paper, an alternative conjunctive numerical model is presented for simulating the overland flow. The numerical model is based on a conjunctive solution of the complete Saint-Venant equations for the surface flow and the two-dimensional Richards equation. Presently available full dynamic models for the overland flow employ classical second-order accurate finite-difference methods for solving the Saint-Venant equations. These methods often result in high frequency oscillations when there are sharp changes in the flow parameters and require ad hoc procedures for obtaining smooth solutions \cite{6}. In this study, a simple to implement high-resolution essentially non-oscillating (ENO) scheme \cite{27} is applied to solve the surface flow component in the conjunctive model. The explicit ENO scheme also results in a simple method for connecting the surface and subsurface flow components. Presently available conjunctive models for the overland flow use Richards equation in pressure-head form, which may result in significant mass balance error for complex subsoil conditions. Many studies \cite{7,9,9} have shown that the 'mixed form' of the Richards equation results in better numerical behaviour than the other forms. Therefore, in this study, the two-dimensional Richards equation in the mixed form is used for simulating the subsurface component. This equation is solved by a recently developed strongly implicit procedure \cite{20}. This is computationally more efficient than the SLOR technique adopted in the currently available conjunctive models. The proposed numerical model is verified using the experimental data available in the literature \cite{29}. Robustness of the numerical model is demonstrated by considering a test case in which a clay lens is present in the subsurface. One of the objectives of developing a sophisticated model is that it can act as a standard of comparison for simplified models. Therefore, the proposed numerical model is used to study the errors that may result in the prediction of the outflow hydrograph if a one-dimensional simulation of the subsurface flow is used.

2 GOVERNING EQUATIONS

Mathematical modeling of overland flow involves the solution of the governing equations for both the surface flow and the groundwater flow with seepage at the ground surface acting as the connecting link. In the present study (Fig. 1), the surface runoff is represented by the one-dimensional flow equations in the x-direction, while the groundwater flow is represented by the two-dimensional Richards equation in the x and z directions.

2.1 Surface flow equations

Surface flow is assumed to occur in a prismatic channel of rectangular section. The one-dimensional shallow water

\[
\text{Rainfall} \rightarrow \downarrow \downarrow \downarrow \text{Surface Flow (1-D)} \rightarrow \text{Infiltration}
\]

\[
\text{Subsurface Flow (2-D)} \rightarrow \text{Ground Surface}
\]

\[
\text{Subsurface Flow (2-D)} \rightarrow \text{Ground Surface}
\]

Fig. 1. Definition sketch for a conjunctive model.
flow equations in conservation form for such a case are given by:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = S$$  \hspace{1cm} (1)

in which,

$$U = \left\{ \begin{array}{c} h \\ q \end{array} \right\}, \quad F = \left\{ \begin{array}{c} q \\ \frac{q^2}{h} + \frac{gh^2}{2} \end{array} \right\}, \quad S = \left\{ \begin{array}{c} (R - I) \\ gh(S_0 - S_f) \end{array} \right\}$$  \hspace{1cm} (2)

In eqns (1) and (2): $h$ = flow depth (m); $q$ = discharge per unit width (m$^2$/s); $g$ = acceleration due to gravity (m/s$^2$); $R$ = volumetric rate of rainfall per unit surface area (m/s); $I$ = volumetric rate of infiltration per unit area (m/s); $S_0$ = bottom slope in the direction of flow; $S_f$ = friction slope; $x$ = distance along the flow direction (m); and $t$ = time (s).

The assumptions and the derivation of the above equations have been reported earlier 36,111 and are not repeated here.

The friction slope, $S_f$ is computed using the Darcy–Weisbach formula taking into account the effect of rainfall on frictional resistance.

$$S_f = f_d \frac{q^2}{8gh^3}$$  \hspace{1cm} (3)

in which, $f_d$ = frictional resistance coefficient. Evaluation of $f_d$ depends on the instantaneous state of flow. The flow is laminar in all the studies reported here and, therefore, $f_d$ is given by the following equation 5.

$$f_d = \frac{C_L}{Re}$$  \hspace{1cm} (4)

where, $Re$ = Reynolds number = $q/\nu$ ($\nu$ = kinematic viscosity of water), and $C_L$ is a constant which depends on the rainfall intensity.

2.2 Subsurface flow equations

In the present study, subsurface flow is modeled as two-dimensional motion of a single-phase incompressible fluid in an incompressible porous medium. The two-dimensional, transient flow equation in an isotropic porous medium is derived by applying the principle of conservation of mass. This equation without sources and sinks within the flow domain can be written as

$$\frac{\partial \theta}{\partial t} + \frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} = 0$$  \hspace{1cm} (5)

where, $\theta$ = volumetric moisture content; $V_x$ and $V_z$ = darcy flow velocity in the $x$ and $z$ direction, respectively, and $x$ and $z$ are distances along the two coordinate directions; $z$ is taken positive downwards. It is assumed that the Darcy's law is applicable for evaluating the velocity components.

$$V_x = -K(\psi)\frac{\partial \psi}{\partial x}, \quad V_z = -K(\psi)\left(\frac{\partial \psi}{\partial z} - 1.0\right)$$  \hspace{1cm} (6)

in which, $\psi$ = pressure head (m) and $K(\psi)$ = unsaturated hydraulic conductivity (m/s) which depends on the pressure head, $\psi$. Substitution of eqn (6) in eqn (5) yields the Richards equation 14:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[K(\psi)\frac{\partial \psi}{\partial x}\right] + \frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z} - 1.0\right)\right]$$  \hspace{1cm} (7)

eqn (7) is said to be in 'mixed form' since it includes both the dependent variables $\theta$ and $\psi$. Hydraulic relationships between the pressure head, $\psi$, and the hydraulic conductivity, $K$, ($\psi-K$ relationship), and between the moisture content, $\theta$, and the pressure head $\psi$ ($\psi-\theta$ relationship), are needed while solving eqns (5) and (6) in the unsaturated zone. In general, $\psi-K$ and $\psi-\theta$ relationships are not unique and soils exhibit different behaviour during wetting and drying phases. This hysteresis in soil characteristics is not considered for the cases studied in the present work. However, the hysteresis can be included by employing different $\psi-K$ and $\psi-\theta$ relationships for wetting and drying processes. Although semi-analytical equations are available for describing $\psi-K$ and $\psi-\theta$ relationships, the soil characteristics derived from the experimental and field data are employed wherever available. These relationships are specific to the case studied and are described later.

3 NUMERICAL SOLUTION

3.1 Surface flow

Surface flow equations constitute a set of nonlinear hyperbolic partial differential equations. A recently developed high-resolution essentially non-oscillating (ENO) scheme for solving the shallow water flow equations 27 is employed in the present study to solve the surface flow equations. The scheme, proposed by Nujic 27, is modified suitably to account for the non-zero source term in the continuity equation. This scheme, unlike many other classical second-order accurate schemes such as the MacCormack method, is non-oscillatory even when sharp gradients in the flow variables are present. The main advantages of this scheme are its simplicity, ease of implementation and ease of extension to a two-dimensional case. It is also very attractive for the present application because it is possible to account for the variable bottom topography in a convenient and accurate way. It is an explicit, two-step predictor-corrector scheme which results in second-order accuracy in both space and time. Only a brief description of the method as applied in the present study is outlined here.

3.1.1 Predictor part

The finite-difference analog of eqn (1) is written here for a finite-difference grid where the subscript $i$ refers to the grid point in the $x$ direction and the superscripts $n$ and * refer to the values at the known time level and the predictor level, respectively. The finite-difference form of eqn (1) for the
explicit determination of \( U_i^t \) is written as

\[
U_i^{t+1} = U_i^n - \Delta t \left[ \frac{F_{i+1/2}^n - F_{i-1/2}^n}{\Delta x} \right] + \Delta t S_i^t
\]  

(8)

where \( F_{i+1/2}^n \) represents the numerical flux through the cell face between nodes \( i + 1 \) and \( i \). \( \Delta x \) is the grid spacing and \( \Delta t \) is the computational time step. All the terms on the right-hand side of eqn (8) are evaluated at the known time level \( n \) and, therefore, \( U_i^t \) can be computed explicitly. The numerical flux \( F_{i+1/2}^n \) is computed using the following formula.

\[
F_{i+1/2}^n = \frac{1}{2} [ F_R + F_L - \alpha(U_R - U_L) ]
\]

in which, \( \alpha = \) a positive coefficient, \( F_R = f(U_R) \) the flux computed using the information from the right side of the cell face and \( F_L = f(U_L) \) = the flux computed using the information from the left side of the cell face. \( U_L \) and \( U_R \) are obtained using a MUSCL (monotone upwind scheme for conservation laws) approach.

\[
U_L = U_i + \frac{\Delta H_i^t}{2}
\]

(10)

\[
U_R = U_{i+1} - \frac{\Delta H_{i+1}^t}{2}
\]

(11)

There are several ways of determining \( \Delta U_i \) and \( \Delta U_{i+1} \) using different slope limiter procedures. The ‘minmod’ limiter is followed in the present study. According to this,

\[
\Delta U_i = \text{minmod}(U_{i+1} - U_i, U_i - U_{i-1})
\]

(12)

where the minmod function is defined as

\[
\text{minmod}(a,b) = \begin{cases} 
  a & \text{if } |a| < |b| \text{ and } ab > 0 \\
  b & \text{if } |b| < |a| \text{ and } ab > 0 \\
  0 & \text{if } ab \leq 0 
\end{cases}
\]

(13)

The positive coefficient \( \alpha \) is determined using the maximum value (for all the grid points) of the largest eigenvalue of the Jacobian of the system of equations. This is given as

\[
\alpha = \max \left( \frac{q_i}{h_i}, \sqrt{g h_i} \right) \quad i = 1 \text{ to } N
\]

(14)

where, \( N \) is the total number of grid points. The vector equation (eqn (8)) gives the predicted values of \( h \) and \( q \) at the unknown time level at any node \( i \).

3.1.2 Corrector part

The vector \( U \) at the unknown time level \( n + 1 \) and at node \( i \), i.e. \( U_i^{n+1} \) is computed using the predicted values and the values at the time level \( n \).

\[
U_i^{n+1} = 0.5 \left[ U_i^n + U_i^t - \Delta t \left( \frac{F_{i+1/2}^n - F_{i-1/2}^n}{\Delta x} \right) + \Delta t S_i^t \right]
\]

(15)

where

\[
F_{i+1/2}^{n+1} = \frac{1}{2} [ F_R^t + F_L^t - \alpha(U_R^t - U_L^t) ]
\]

Following the recommendations of Alcrudo et al., \( U_R^t \) and \( U_L^t \) are determined from \( U_i^{n+1} \) and \( U_i^t \) using the same \( \Delta U_i \) and \( \Delta U_i^t \) as determined in the predictor step. This procedure results in better numerical stability.

In eqn (15), only the source term in the momentum equation (the second component of the vector equation) is evaluated using the predicted values of \( h \) and \( q \). However, the source term in the continuity equation i.e. \( \left( R - I_i \right) \) is evaluated using the values at the known time level instead of the predicted values. Strictly speaking, this procedure decouples the subsurface flow computations and the surface flow computations during the computational time step \( \Delta t \). However, the response of the subsurface flow to the variation in surface flow depth is much slower than the response of surface flow to changes in the rate of infiltration. Therefore, the above decoupling does not affect the results significantly. In fact, numerical experimentation showed that determination of the infiltration rate, \( I_i \), during the corrector step by using the predicted flow depth did not alter the results. On the other hand, the decoupling procedure resulted in significant savings of the computational time, since the subsurface flow is computed only once during a time step.

3.2 Initial and boundary conditions

Values of the flow depth, the discharge and the infiltration rate are specified at all the nodes at time \( t = 0 \) as the initial conditions. The initial infiltration rate is equal to the rainfall rate. Although the initial flow depth and the discharge are equal to zero (overland flow on an initially dry surface), a very thin water film of depth \( h_{ini} \) and corresponding uniform flow discharge, \( q_{ini} \) are assumed to exist at time \( t = 0 \). This assumption is made to overcome the numerical singularity in a simple way. However, it should be noted that the outflow hydrograph may be sensitive to the \( h_{ini} \) value and, therefore, it should be chosen as small as possible.

The explicit finite-difference scheme described earlier can be used to determine \( h \) and \( q \) at the unknown time level only at the nodes \( i = 2 \) to \( N - 1 \). The values of the variables at the upstream and the downstream ends of the domain are determined using the appropriate boundary conditions. The discharge at the upstream end is equal to zero. However, the discharge at the upstream end is specified as \( q_{ini} \) to be consistent with the initial conditions. The flow depth at the upstream end can then be determined using the negative characteristic equation. In the present study, a simple extrapolation procedure is adopted to determine the upstream flow depth from the depth at the interior nodes. Numerical experimentation in the initial stages of the model development showed that the extrapolation procedure gave satisfactory results. Similar extrapolation procedure is adopted to determine \( h \) and \( q \) at the last node \( N \).
3.3 Stability condition

The high-resolution Lax–Friedrichs scheme adopted in the present study is an explicit scheme. Therefore, the computational time step, $\Delta t$, is chosen using the CFL condition.

$$C_n = \frac{\Delta t}{\Delta x} \left( \frac{\rho}{h} + \sqrt{\rho k} \right) < 1.0$$  \hspace{1cm} (17)

in which $C_n = \text{Courant number}$. $\Delta t$ is chosen dynamically in the numerical model such that eqn (17) is satisfied at all the nodes $i = 1, 2, \ldots, N$.

3.4 Subsurface flow

In order to determine the infiltration rate $I$ in the continuity equation for the surface flow, the subsurface flow equations have to be solved along with an appropriate boundary condition at the ground surface. In the present study, a recently developed strongly implicit finite-difference scheme for the mixed formulation of the Richards equation is used to simulate the unsaturated subsurface flow conditions. This scheme ensures mass balance in its solution regardless of time step size and nodal spacings, and has no limitations when applied to field problems. It is also easy to incorporate different types of boundary conditions in this scheme. In the present study, two different models 1DS2DSS and 1DS1DSS are developed for simulating the overland flow. In the model 1DS2DSS (one-dimensional surface flow with two-dimensional subsurface flow), the infiltration rate, $I$, is determined by solving the two-dimensional Richards equation. In the model 1DS1DSS (one-dimensional surface flow with one-dimensional subsurface flow), the subsurface flow is assumed to occur only in the vertical direction. Fig. 2 shows the definition sketch for the model 1DS1DSS. In this model, the infiltration rate at any distance $x$ is determined using the one-dimensional Richards equation with the surface flow depth at that point as the top boundary condition. Model 1DS1DSS results in significant savings of CPU time. Numerical solution of the Richards equation is described in the following section.

The subsurface flow domain is divided into a number of rectangular blocks. The moisture content, $\theta$, and the pressure head, $\psi$, are specified at the center of the block (the node), while the velocities are specified at the interblock faces. The subscript $i$ refers to the block number in the $x$ direction and the subscript $j$ refers to the block number in the $z$ direction. The superscripts $n$ and $n + 1$ refer to the known and the unknown time levels, respectively. The finite difference form of the eqn (5) is

$$\frac{\theta_{i,j}^{n+1} - \theta_{i,j}^n}{\Delta t} + \frac{(V_{i+1,j} - V_{i,j})}{\Delta x} + \frac{(V_{i,j+1} - V_{i,j-1})}{\Delta z} = 0$$  \hspace{1cm} (18)

where the bar is used to denote the time-averaged value of the velocity. $\Delta x$ and $\Delta z$ are the nodal spacings in the $x$ and $z$ directions, respectively. The time-averaged velocities are determined by

$$\bar{V} = wV^{n+1} + (1 - w)V^n$$  \hspace{1cm} (19)

in which, $w = \text{time weighting factor}$. The velocity at any interblock face is determined using the pressure heads at the neighbouring cell centers. For example;

$$V_{i,j+1/2} = -K_{j+1/2}(\psi_{i,j+1} - \psi_{i,j}) - \Delta z \frac{\Delta \psi}{\Delta z}$$  \hspace{1cm} (20)

in which, $K_{i,j+1/2}$ is the unsaturated hydraulic conductivity evaluated at the interblock faces between the nodes $(i, j + 1)$ and $(i, j)$. Substitution of eqns (19) and (20) in eqn (18) yields

$$\frac{w \Delta t}{\Delta x} \left[ -K_{i+1/2,j}^{n+1}(\psi_{i+1,j}^{n+1} - \psi_{i,j}^{n+1}) \right. \\
\left. + K_{i-1/2,j}^{n+1}(\psi_{i,j+1}^{n+1} - \psi_{i-1,j}^{n+1}) \right] + \frac{w \Delta t}{\Delta z} \left[ -K_{i,j+1/2}^{n+1}(\psi_{i,j+1}^{n+1} - \psi_{i,j}^{n+1} - \Delta z) \right. \\
\left. + K_{i,j-1/2}^{n+1}(\psi_{i,j}^{n+1} - \psi_{i,j-1}^{n+1}) \right] + \theta_{i,j}^{n+1} \\
- \left[ \frac{\theta_{i,j}^n - (1 - w) \Delta \psi}{\Delta x} (V_{i+1,j}^{n+1} - V_{i,j}^{n+1}) \right] \\
\left. - (1 - w) \Delta t \frac{\Delta \psi}{\Delta z} \right] = 0$$  \hspace{1cm} (21)

The unsaturated hydraulic conductivity at an interblock face is estimated using the pressure heads at the neighbouring cell centers. Cooley, Haverkamp and Vauclin and Narasimhan and Witherspoon have suggested various ways for this purpose. Haverkamp and Vauclin state that the geometric mean is the best choice for estimating the interblock conductivities. However, Hong et al. reported (also experienced during the development of the present model) that the iterative solution of eqn (21) fails to converge if the above procedure is adopted for estimating the $K$ value. This is especially true for infiltration into initially very dry soils. The geometric mean is strongly weighted towards the lower value and, therefore, water cannot drain easily if the soil is initially dry. This results in a non-physical build up of pressure. In this study, the interblock hydraulic conductivity is estimated by the...
for such a situation is the specification of the flux equal to the rainfall rate. The top boundary condition affects the accuracy of the results.

3.7 Boundary conditions

For subsurface flow resulting from rainfall infiltration, the top boundary condition changes with time. During the initial stages of the rainfall, there is no ponding and the infiltration rate is equal to the rainfall rate. The top boundary condition for such a situation is the specification of the flux equal to the rainfall rate. As time progresses, the upper layers of the subsoil become saturated and then infiltration rate starts decreasing. Before starting the solution of the Richards equation for any time step, the flux at the top boundary is estimated by taking \( \psi_b = 0 \). If this velocity is greater than the rainfall rate, then the flux type of boundary condition is applied. Otherwise, a head type of boundary condition \( (\psi_b(x) = h(x) \text{ water flow depth at that point}) \) is applied. The time to ponding comes out as a part of the solution.

Specified head or specified flux conditions can be imposed at the lateral and bottom boundaries. For the physical situations considered in the present study, a no-flux boundary condition is imposed at the right and left boundaries. The \( \psi \) values at the bottom boundary are obtained using a simple extrapolation from the interior points. This approximation does not introduce errors because the bottom boundary is taken fairly deep and the moisture front does not reach that far for the computational times considered.

4 VERIFICATION OF THE MODEL

The model presented in the previous section is tested with the experimental data of Smith and Woolhiser 29 for a conjunctive surface–subsurface flow. The experiments 29 were conducted in a soil flume of 12.2 m length, 51 mm wide and 1.22 m deep. The slope of the channel was 0.01. The fluid used was a light oil whose viscosity \( \nu = 1.94 \times 10^{-4} \text{ m}^2/\text{s} \). A rainfall simulator was used to produce a rainfall of intensity 250 mm/h over a duration of 15 min. The flow regime is laminar throughout and the \( C_L \) value 5 in eqn (4) is taken as 92.0. The soil used was Poudre fine sand and it was placed in three layers of differing bulk densities. Thicknesses of the three layers, 1, 2 and 3, were 76.5, 229.5 and 761 mm, respectively. The hydraulic properties (\( \theta \) vs. \( \psi \), and \( K \) vs. \( \psi \)) curves of the three layers were determined experimentally 29. The relative hydraulic conductivity, \( K_r \), and the effective saturation, \( S_e \), are defined as

\[
K_r = \frac{K}{K_s} \quad (23)
\]

\[
S_e = \frac{(\theta - \theta_r)}{\theta_s - \theta_r} \quad (24)
\]

in which, \( K_s \) and \( \theta_r \) are the saturated hydraulic conductivity and saturated moisture content, \( \theta_s \) is the residual moisture content. The curves for \( S_e \) vs. pressure head and \( K_r \) vs. pressure head (Figs. 4 and 5 of Smith and Woolhiser 29) are represented in the present study using the following equations.
Conjunctive surface–subsurface modeling of overland flow

\[
K_r \begin{cases} 
\left( \frac{\psi}{\theta} \right)^n & \text{if } \psi \leq -13.729 \text{ cm} \\
0.5228 + 0.0204 \psi - 0.0009 \psi^2 & \text{if } -13.729 \leq \psi \leq -1.0 \text{ cm} \\
0.5015 & \text{if } \psi > -1.0 \text{ cm}
\end{cases}
\] (26)

Layer 2:

\[
S_e = \begin{cases} 
\left( \frac{\psi}{\theta} \right)^\lambda & \text{if } \psi \leq -17.196 \text{ cm} \\
0.9163 + 0.0011 \psi - 0.00012 \psi^2 & \text{if } -17.196 \leq \psi \leq -2.0 \text{ cm} \\
0.9093 & \text{if } \psi > -2.0 \text{ cm}
\end{cases}
\] (27)

\[
K_r \begin{cases} 
\left( \frac{\psi}{\theta} \right)^n & \text{if } \psi \leq -16.784 \text{ cm} \\
0.6229 + 0.0134 \psi - 0.0013 \psi^2 & \text{if } -16.784 \leq \psi \leq -1.0 \text{ cm} \\
0.6082 & \text{if } \psi > -1.0 \text{ cm}
\end{cases}
\] (28)

Layer 3:

\[
S_e = \begin{cases} 
\left( \frac{\psi}{\theta} \right)^\lambda & \text{if } \psi \leq -25.18 \text{ cm} \\
0.9642 + 0.01 \psi - 0.000536 \psi^2 & \text{if } -25.18 \leq \psi \leq -2.0 \text{ cm} \\
0.962 & \text{if } \psi > -2.0 \text{ cm}
\end{cases}
\] (29)

\[
K_r \begin{cases} 
\left( \frac{\psi}{\theta} \right)^n & \text{if } \psi \leq -21.34 \text{ cm} \\
0.7387 + 0.0063 \psi - 0.0011 \psi^2 & \text{if } -21.34 \leq \psi \leq -1.0 \text{ cm} \\
0.7313 & \text{if } \psi > -1.0 \text{ cm}
\end{cases}
\] (30)

The values of \( K_r, \theta_s, \theta, \lambda, \eta \) and \( \psi_b \) (hypothetical bubbling pressure potential) for the three soil layers are given in the Table 1.

The numerical model is run for a length of 15 m with 2.8 m acting as the buffer length for application of the downstream boundary condition. The finite-difference grid spacings in the \( x \) and \( z \) directions, \( \Delta x \) and \( \Delta z \), are equal to 0.25 m and 12.7 mm, respectively, and Courant number \( C_n = 0.65 \). A very small value of 0.2 mm is taken as the initial flow depth, \( h_{in} \). The numerical parameters for the subsurface flow computations are: \( w = 1.0, \gamma = 0.5 \) and \( \epsilon = 10^{-5} \). Although, only a high-resolution (ENO) scheme for the surface flow is presented in this paper, the surface flow equations were also solved using the classical MacCormack scheme \(^{11} \) as a double check. Both the methods gave essentially the same results for this run.

Fig. 3 shows the runoff hydrograph at the downstream end \( (x = 12.25 \text{ m}) \) for the above experiment obtained (i) experimentally \(^{29} \), (ii) numerically by Akan and Yen \(^{5} \), (iii) numerically by the model 1DS1DSS and (iv) numerically by the model 1DS2DSS. It is clear from this figure that both the models 1DS1DSS and 1DS2DSS give almost the same results. Model 1DS2DSS shows only some marginal effect of two-dimensional subsurface flow. It can be seen that the numerical results obtained by the models 1DS1DSS and 1DS2DSS compare satisfactorily with the experimental as well as with the earlier numerical results of Akan and Yen \(^{5} \). The present numerical results are lagging as far as the rising limb of the hydrograph is concerned. However, they are better for the recession phase. The discrepancy between the present numerical results and the earlier numerical results of Akan and Yen \(^{5} \) could be due to minor differences in processing the soil-moisture characteristic curves as given by Smith and Woolhiser \(^{29} \). It should be noted here that the results for the outflow hydrograph are sensitive to the hydraulic properties of the subsurface soil. A 2.8% decrease in the \( K_r \) value for layer 1 and a corresponding 2.8% increase in the \( K_r \) value for layer 2 resulted in an outflow hydrograph which matched very well with the experimental data. Fig. 4 shows the volumetric saturation profile under the ground along the depth at a distance of 5.5 m from the upstream end at different times. This figure shows the present numerical results and the previous numerical results obtained by Smith and Woolhiser \(^{29} \). No attempt is made to compare the numerical results with the experimental data because the data points are very few in number. It can be seen from Fig. 4 that the present and previous numerical results of Smith and Woolhiser \(^{29} \) match satisfactorily at early times. However, as time progresses, the wetting front in the present model moves faster than the wetting front in Smith and Woolhiser's \(^{29} \) model.

### Table 1. Hydraulic properties of the soil

<table>
<thead>
<tr>
<th>Layer</th>
<th>( K_r ) (cm/day)</th>
<th>( \theta_s )</th>
<th>( \theta )</th>
<th>( \lambda )</th>
<th>( \eta )</th>
<th>( \psi_b ) (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>566.9</td>
<td>0.460</td>
<td>0.020</td>
<td>2.03</td>
<td>8.09</td>
<td>-10.0</td>
</tr>
<tr>
<td>2</td>
<td>365.7</td>
<td>0.504</td>
<td>0.025</td>
<td>2.20</td>
<td>8.60</td>
<td>-13.0</td>
</tr>
<tr>
<td>3</td>
<td>268.2</td>
<td>0.543</td>
<td>0.025</td>
<td>2.31</td>
<td>8.93</td>
<td>-17.0</td>
</tr>
</tbody>
</table>
Figure 4. Verification of models 1DS1DSS and 1DS2DSS: volumetric saturation in subsurface.

Because the computed infiltration rate is larger. This is consistent with the results presented in Fig. 3, where the outflow hydrograph by the present model is lagging the previous results. Fig. 4 also shows that the two-dimensional subsurface flow effects are marginal for the particular case studied here.

Sensitivity of the numerical results to the numerical parameters $h_{ini}$ and the weighting coefficient, $\gamma$, is studied using the model 1DS1DSS. Fig. 5 shows the effect of ad hoc specification of the initial flow depth, $h_{ini}$. Initial flow depth in these runs varied from 0.2 to 0.5 mm. The numerical results for the rising limb of the hydrograph are sensitive to the specified $h_{ini}$ value. However, there is no effect of the $h_{ini}$ on the hydrograph beyond a certain time (11 min). Although it appears that better results can be obtained by taking $h_{ini} = 0.5$ mm, this value should be chosen as small as possible in order to faithfully represent the initial dry conditions. The effect of weighting coefficient, $\gamma$ (in the determination of the interblock hydraulic conductivity), on the outflow hydrograph is shown in Fig. 6. It is obvious from Fig. 6 that the numerical results are highly sensitive to the $\gamma$ value. Hong et al. recommended a value of 0.5 for $\gamma$, which results in equal weightage to higher and lower values of $K$ on the two sides of the face. In an earlier study, Haverkamp and Vaucelin concluded that the interblock conductivity should be weighted more towards the

![Figure 5. Effect of $h_{ini}$ on the overland flow hydrograph.](image-url)
lower value of $K$ and, accordingly, they used the geometric mean. In the limit of grid convergence, $\gamma = 0.35$ gave better results than $\gamma = 0.5$ for the test case studied here. However, the above conclusion cannot be generalized because it is based on a comparison of numerical results with experimental data which may have uncertainties in them. This aspect needs further study.

Often soil deposits are encountered in the field (Ex: Alluvial deposits) which are more or less homogeneous except for the presence of an isolated clay lens. This clay lens acts as a barrier to the vertical movement of the wetting front in the ground, and can significantly affect overland flow. In such a case, sharp gradients may occur in the overland flow hydrograph, and models which use traditional numerical methods result in numerical oscillations. This is demonstrated by considering a case (Fig. 7) in which a clay lens of length 4 m is present at a depth of 1.14 cm from the surface. The subsoil characteristics for this run were same as those for layer 1 in the experiments by Smith and Woolhiser 29. Other input parameters, such as the rainfall rate and length of the domain, were also same. The outflow hydrographs for this case, obtained using (i) the model 1DS2DSS with ENO scheme, and (ii) the model 1DS2DSS with a traditional finite-difference scheme, such as the MacCormack method, are compared in Fig. 8. A Courant number of 0.8 was used for the ENO scheme while a value of 0.75 was used for the MacCormack scheme. It is very clear from Fig. 8 that the numerical results obtained using the ENO scheme are superior to those obtained using the MacCormack scheme. A Courant number value as low as 0.2 was required to suppress the numerical oscillations in the MacCormack scheme. This is also the order of Courant number value used by Zhang and Cundy 37 in some of their simulations. It should be noted here that low Courant number values result not only in higher computational times, but also in diffusive errors 11. Numerical results for this test case also demonstrate that the model 1DS2DSS can be used to simulate the effect of complex subsurface conditions on an overland flow hydrograph. Immediately after the arrival of the wetting front at the clay lens, the infiltration rate for that part of the channel which is directly above the clay lens becomes negligible, and there is a sharp rise in the outflow hydrograph. Model 1DS2DSS was able to simulate this effect.

5 SIMPLIFIED MODELING

One of the objectives of developing a sophisticated model is that it can act as a standard of comparison for simplified models. Some of the currently available conjunctive models for the overland flow 2,29 assume that the subsurface flow in the unsaturated zone is predominantly one-dimensional. Therefore, a one-dimensional Richards equation is used in their models. Govindaraju and Kavvas 16 have demonstrated how such an assumption is not valid if the groundwater table is close to the surface. Herein, the model 1DS2DSS is used to study the error in the prediction of the outflow hydrograph using a one-dimensional simulation of the subsurface flow for the case of a deep groundwater table. Three different hypothetical cases, as represented in Fig. 9, are considered for this purpose. In this figure, the soil characteristics of layers 1, 2 and 3 are same as the soil characteristics for layers 1, 2 and 3 in the experiments by Smith and Woolhiser 29. The same surface flow conditions (length = 15 m, slope = 0.01, $v = 1.94 \times 10^{-3} \text{ m}^2/\text{s}$, $R = 250 \text{ mm/h}$, $C_L = 92.0$) are used in these simulation runs. The numerical simulations are performed using $\Delta x = 0.25 \text{ m}$, $h_{in} = 2 \times 10^{-3} \text{ m}$, $\alpha = 1.0$ and $\gamma = 0.5$, in both models, 1DS1DSS and 1DS2DSS. In Fig. 10, the outflow hydrograph for the case shown in
Fig. 8. Outflow hydrograph for case in Fig. 7: ENO scheme vs. MacCormack scheme.

Fig. 9(a) is compared with the outflow hydrograph for Smith and Woolhiser's 29 experiment. Both the hydrographs are obtained using the model 1DS1DSS. It can be observed from Fig. 10 that the ad hoc simplification of layered soil conditions in Smith and Woolhiser’s 29 experiment by a single layer, as given in Fig. 9(a), affects the outflow hydrograph significantly. This is because the spatial variation in soil characteristics in the vertical direction affects the infiltration rate. Appropriate homogenization procedures should be adopted if simple analytical solutions for infiltration based on homogeneous soil characteristics are to be used while simulating overland flow on layered subsoil. In the absence of appropriate homogenization, model 1DS1DSS can be used to study the effect of subsoil layering on the outflow hydrograph. Application of the computationally more intensive model 1DS2DSS is not required if there is only vertical layering in the subsoil. Both models, 1DS1DSS and 1DS2DSS, gave essentially the same results (not shown here) for the case shown in Fig. 9(a). For this case of uniform rainfall rate, the whole surface length is ponded at the same time, and the wetting front moves essentially in the vertical direction. The two-dimensional subsurface flow effects are only marginal and models 1DS1DSS and 1DS2DSS give similar results. Model 1DS1DSS is also run for Smith and Woolhiser's 29 experiment by assuming that the infiltration rate is uniform throughout the length, as schematically represented in Fig. 11(a). Such an assumption is valid if the surface flow depth does not have a significant effect on the infiltration rate. Fig. 11(b) compares the outflow hydrograph from this run with the previous outflow hydrograph obtained using the general 1DS1DSS model. It can be clearly seen that both the hydrographs are essentially the same. This indicates that the infiltration rate is more or less uniform throughout the length as long as there is no spatial variability of soil characteristics in the x direction. The non-uniform surface flow depth does not have a significant effect on the infiltration rate. This assumption of uniform infiltration rate greatly reduces the computational time (as much as by 10 times) while running the model 1DS1DSS and, therefore, is attractive from an engineering point of view.

The outflow hydrograph for the case shown in Fig. 9(b) is considered for study if the two-dimensionality effects in the subsurface flow become significant due to the spatial variation of the soil characteristics in the x direction. The outflow hydrographs for this case obtained using models 1DS1DSS and 1DS2DSS are shown in Fig. 12. It can be clearly seen from this figure that, even in this case, the two-dimensional

Fig. 9. Schematic representation of subsurface conditions considered in the numerical simulation studies.
Conjunctive surface–subsurface modeling of overland flow

Fig. 10. Effect of simplification of layered subsoil conditions using a single layer on outflow hydrograph.

effects are only marginal and the application of the simple model 1DS1DSS gives satisfactory results. The models developed in the present work are also applied to simulate the outflow hydrograph for the case shown in Fig. 9(c), in which the subsurface soil characteristics vary in a complicated manner. It should be noted here that even the model 1DS1DSS considers the heterogeneity of the subsurface and there is no homogenization. The only difference between models 1DS1DSS and 1DS2DSS is that in the former case the flow is assumed to be only in the vertical direction. The outflow hydrographs obtained using models 1DS1DSS and 1DS2DSS for the above case are shown in Fig. 13. The effect of two-dimensionality in the subsurface flow is very much evident in this case. The one-dimensional model shows an early ponding and a reduced peak. It can be concluded from the above simulation studies that, in many cases where the groundwater table is very deep, the unsaturated flow in the ground may be considered one-dimensional for the purpose of simulating the overland flow. This assumption is valid, especially when the spatial variation in the soil characteristics is only in the vertical direction. However, two-dimensionality comes into the picture if the spatial variation of the soil characteristics is very complex and is in both x and z directions. Model 1DS2DSS should be used in such cases.

Fig. 11. Effect of simplification of model 1DS1DSS using the assumption of uniform infiltration rate.
6 CONCLUSIONS

In this study, a conjunctive surface–subsurface numerical model is developed for the simulation of overland flow. In this model, the complete Saint-Venant equations are solved using a simple, explicit, high-resolution essentially non-oscillating (ENO) finite-difference scheme. The subsurface flow component in the proposed model is represented by the two-dimensional Richards equation in the mixed form. This equation is solved by a recently developed efficient strongly implicit finite-difference scheme. The proposed conjunctive model is verified using experimental data available in the literature. The ability of the proposed model to handle complex subsurface conditions is demonstrated through the numerical simulation studies. It is shown that the proposed model does not result in high-frequency numerical oscillations even for hydrographs with steep rising limbs. It is found from the numerical simulation studies that, in many cases where the groundwater table is deep, the unsaturated flow in the ground may be considered one-dimensional. It is also possible to obtain further simplification if it is assumed that the infiltration rate is more or less uniform. Such an assumption may be valid if the spatial variation in the soil characteristics is only in the vertical direction. However, two-dimensional effects do become significant if the spatial variation of the soil characteristics is complex and is in both \( x \) and \( z \) directions.

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Conjunctive surface–subsurface modeling of overland flow


