Technical Note

Computation of a dam-break flood wave in channel transitions

P. K. Mohapatra & S. Murty Bhallamudi*

Department of Civil Engineering, Indian Institute of Technology, Kanpur 208016, India

(Received 13 July 1994; revised received 21 June 1995; accepted 26 September 1995)

Dam-break flow in channel transitions is simulated by numerically solving the system of governing two-dimensional equations. A simple algebraic coordinate transformation technique is adopted for converting the general physical domain of a transition into a rectangular computational domain. The MacCormack scheme is used for solving the transformed equations. Numerical results of the present study show satisfactory agreement with the experimental data available in the literature. They also match previous numerical results well. Effects of the contractions and expansions on the dam-break flow depth at the dam site is also studied.

Key words: unsteady two-dimensional flow, dam-break flow, coordinate transformation, MacCormack scheme, transitions.

INTRODUCTION

The analysis of dam-break flow is a part of dam design and safety analysis and is considered a vital task in hydraulic engineering practice. An engineer is interested in the estimation of peak-flood flow, corresponding maximum flow depth and its time of occurrence at a specified location in the path of flood wave. One is also interested in knowing the depth of flow at the dam site resulting from the dam-break as related to upstream and downstream flow depths prior to the failure.

One-dimensional models, whether analytical or numerical, cannot be used when the dam is located in a channel transition. Marshall and Menendez and Martin used radial flow theory to study dam-break flow in rectangular channels with contracting and diverging side walls, respectively. Townson and Al-Salihi used the method of characteristics and the radial coordinate system to simulate dam-break flow for combinations of parallel, converging and diverging boundaries both upstream and downstream of the dam-break. However, the radial flow method is strictly applicable only for dam-break flows in straight walled transitions. Dam-break flow in a general transition with curved boundaries can be simulated satisfactorily by using a two-dimensional numerical model. Bellos et al. used a method which is a combination of the finite-element and finite-difference methods. Garcia Navarro and Alcrudo solved the same problem using a TVD scheme with a finite-volume approach.

Bhallamudi and Chaudhry simulated steady supercritical flows in channel transitions using a simple algebraic coordinate transformation and a false transient approach. In this short note, we demonstrate how the above model can also be applied to simulate the transient dam-break flows by a simple modification of the initial and boundary conditions. The numerical and experimental data of Townson and Al-Salihi are used for verification of numerical results.

GOVERNING EQUATIONS

The physical domain of a channel transition may be transformed to an appropriate computational domain for easier application of finite-difference methods. In this study, the physical domain of the channel transition
respectively; $b'(\xi)$ denotes the derivative of $b(\xi)$ with respect to $\xi$; $h$ is the flow depth; $u$ is the depth averaged flow velocity in the $x$-direction; $v$ is the depth averaged velocity in the $y$-direction; $t$ is the time; $g$ is acceleration due to gravity; $S_{0x}(x,y)$ and $S_{f(x,y)}$ are the channel bottom slope and the friction slope in the $x$- and $y$-directions, respectively. $S_{0x}$ and $S_{f}$ are calculated using the empirical Manning equation. Equation (3), presented above, is based on the two-dimensional shallow water theory. Therefore, the model presented here should not be used in case of large-angle expansions. Also, the model cannot be applied when the boundaries of the flow region are modified over time.

**NUMERICAL SCHEME**

The governing equations are numerically solved in this work by the MacCormack finite-difference scheme. MacCormack scheme is an explicit, second order accurate, shock-capturing scheme. Details of this scheme as applied to a transition problem are given elsewhere and are not repeated here. Specific to the present study are initial and boundary conditions and these are discussed below.

**Initial conditions**

To start the unsteady state computations, the values of all the dependent variables $u$, $v$ and $h$ at time $t = 0$ have to be specified at all the grid points. In the present application, $u$ and $v$ at all the grid points are equal to zero at time $t = 0$ since there is no flow prior to the dam-break. The flow depth, $h$, is specified below:

(i) **Upstream of the dam.** Flow depth is equal to the flow depth in the upstream reservoir.

(ii) **Downstream of the dam.** There are two types of conditions downstream of the dam viz dry-bed condition and the wet-bed condition. In the latter case, the actual depth existing at $t = 0$ is specified as the initial condition. For the dry-bed condition, zero flow depth can not be used due to singularity problems. Therefore, a very small non-zero value for flow depth is specified as the initial condition. In this study, a value of $h_d = 10^{-3} \text{ m}$ is used as the initial condition.

**Boundary conditions**

The solid side wall and symmetry boundaries are treated by the reflection technique in order to implement the condition that the normal velocity is equal to zero. For open boundaries, i.e. inflow and outflow boundaries,
specification of boundary conditions depends upon the flow characteristics.14,17

Inflow boundary

Three boundary conditions need to be specified at the inflow section if the flow is super-critical at this point. However, only two conditions need to be specified if the flow is sub-critical. For the cases studied here, the solid wall at the upstream end of the reservoir forms the inflow boundary. The flow here is sub-critical as there is no inflow through the solid boundary. Therefore, the velocities, \( u \) and \( v \), at this boundary are specified as zero for all times \( t \) and these constitute the actual boundary conditions. The flow depth \( h \) at this boundary should then be computed using the governing equations. The MacCormack scheme cannot be applied at the boundary because of grid requirements. Characteristic methods may be applied for this purpose. However, such methods are complicated and hence a simple approximate procedure is adopted in this study. The finite-difference scheme can be applied at all the interior grid points, i.e., for \( i = 2, \ldots, N - 1 \) (\( N \) is the last grid point in the \( \xi \)-direction) in order to march the solution from time level \( k \) to time level \( k + 1 \). The already determined values of \( h \) at the time level \( k + 1 \) at nodes 2 and 3 are then used to determine the flow depth at node 1, i.e., the upstream section. The following linear extrapolation equation is used.

\[
h_{1,j}^{k+1} = 2h_{2,j}^{k+1} - h_{3,j}^{k+1}
\]

The above extrapolation procedure is used in both the predictor and corrector parts of the MacCormack scheme. Theoretically speaking, the above method gives exact results as long as the dam-break wave does not arrive at the upstream location since the initial steady-state water level is the same at all points in the reservoir. Beyond this time, some errors are introduced. For the cases studied here, however, these errors are insignificant.

Outflow boundary

No boundary condition needs to be specified at the outflow boundary if the flow is super-critical here. On the other hand, one boundary condition has to be specified at this point if the flow is sub-critical. This is usually in the form of either a hydrograph or a rating curve. The other variables may be determined using either the more accurate characteristic method or the approximate extrapolation procedure as outlined earlier. As far as the present study is concerned, the dam-break wave does not arrive at the downstream section during the period for which the computations are made. This is made possible by taking a sufficiently long downstream channel. Therefore, the boundary conditions at the downstream end are the same as the initial conditions at that location.

RESULTS AND DISCUSSION

Model verification

The numerical model presented in the previous section is applied to four different transition layouts shown in Fig. 2. Also, two different downstream conditions: (i) the dry-bed condition and (ii) the wet-bed condition are considered. Computations are made for the following input values which correspond to the experiments conducted by Townson and Al-Salihi16 L (total length of the channel) = 4 m; \( L_u \) (length of the channel upstream of the dam) = 1.8 m; \( L_d \) (length of the channel downstream of the dam) = 2.2 m; \( \theta_o \) (angle of the expansion) = 5°; \( \theta_i \) (angle of the contraction) = 5°; \( B_d \) (width of the channel at the dam location) = 0.1 m; \( h_u \) (initial flow depth on the upstream side of the dam) = 0.1 m; and \( h_d \) (initial flow depth on the downstream side of the dam) = 0.00001 m for the dry-bed condition and 0.0176 m for the wet bed condition. A finite-difference grid of \( \Delta \xi = 0.05 \) m and \( \Delta \eta = 0.0143 \) is used in the numerical simulation. The channel is assumed to be horizontal and frictionless. The computational time step \( \Delta t \) is determined using the CFL condition for stability in which the Courant number is taken equal to 0.85.

Numerical and experimental wave profiles, 1.5 s after the instantaneous dam-break for the case of the dry downstream bed condition, are shown in Fig. 3. The wave due to dam-break has not arrived at the upstream boundary until this time. Figures 3(a) and 3(d) show good agreement between the numerical and experimental results. The present numerical results also match very well with the numerical results obtained by Townson and Al-Salihi.16 Figures 4(a) to 4(d) show the numerical and experimental water surface profiles in the four transitions with finite initial downstream depth. The profiles are taken 2.5 s after the instantaneous dam-break. In this case, the wave due to dam-break has certainly arrived at the upstream end as indicated by a decrease in the flow depth at that location. It can be observed from Fig. 4 that the agreement between the experimental and numerical results is satisfactory for the
case of the wet downstream bed condition also. The above results verify the numerical model and the approximate treatment of the boundary conditions presented in the previous sections. The use of the characteristic method requires the inclusion of a shock-fitting method to simulate the movement of the wave front. The wave front velocity has to be determined explicitly at every instant to advance the solution. This is not required in the application of shock-capturing finite-difference methods. The location of the wave front comes out as a result. The results shown in Figs 3 and 4 indicate that the movement of the wave front is simulated adequately by the present numerical model.

Flow depth at the front

The controlling influence of transition on the dam-break flow depth is studied by plotting steady proportionate front height \((h_0 - h_d)/h_u\) as a function of initial depth ratio \(h_d/h_u\) in Figs 5(a)–5(d). In the above, \(h_0\) is the flow depth at the front after the dam-break. These figures include the classical Stoker solution, experimental results and present numerical results. In all the four cases, the numerical results matched well with the experimental results. Stoker’s analytical solution is valid only for the case of dam-break flow in straight prismatic channels. Therefore, the numerical and the experimental results are close to the Stoker solution for the first two cases where the downstream channel is straight. However, the numerical, as well as the experimental, results are significantly lower than the Stoker solution for the last two cases where the downstream channel is expanding. This indicates the controlling influence of the downstream expansion.

Sub-critical and super-critical flow regions

Flow changes from sub-critical to super-critical and back as the dam-break flow occurs. The boundary between the sub-critical and super-critical flow regions
Computation of a dam-break flood wave in channel transitions

Fig. 4. Water surface profile at $t = 2.5$ s after the dam-break (initial downstream depth: $0.0176$ m): (a) parallel–parallel; (b) converging-parallel; (c) converging-diverging; (d) parallel-diverging.

depends on the water levels on the upstream and downstream sides of the dam prior to the dam-break. This also depends on the time from the instant the dam breaks. This can be studied by calculating the Froude number at every cross-section. Three different cases of dam-break flow in a transition of the type shown in Fig. 2(d) are considered for illustration. The downstream flow depth prior to the dam-break is taken equal to $0.2$ m. The flow depth in the upstream reservoir is equal to $1.2$, $2.2$ and $3.2$ m for the three cases, respectively. The width of the upstream reservoir is equal to $1$ m and the length of the reservoir is equal to $10$ m. The angle of expansion is equal to $5^\circ$. Results for Case 1 ($h_u = 1.2$ m) indicated that the flow is sub-critical throughout. The wave front is at a distance of $11.2$ m and $14.2$ m at $t = 0.5$ s and $1.5$ s, respectively. Results for Case 2 ($h_u = 2.2$ m) at $t = 0.5$ s showed a transition from sub-critical to super-critical at a distance of $0.2$ m downstream of the dam location. There is a transition back to subcritical flow at a distance of $1.0$ m from the dam location and this is accompanied by a weak hydraulic jump. The wave front is $2.0$ m downstream of the dam. As time progresses, the first transition point from subcritical to super-critical moves upstream and the second transition point from super-critical to sub-critical moves downstream. The hydraulic jump becomes more prominent. At time $t = 1.5$ s, the hydraulic jump is located $2.0$ m downstream of the dam and the wave front is at $6.0$ m. Results for Case 3 ($h_u = 3.2$ m) indicated that the first transition from sub-critical to super-critical occurs on the upstream side of the dam and this moves further upstream as time progresses. The second transition from super-critical to sub-critical occurs right at the wave front and there is no formation of the hydraulic jump. It may be concluded from these results that the flow remains sub-critical if the difference between the upstream ($h_u$) and downstream ($h_d$) flow depths prior to the dam-break is small. There is a transition from subcritical to super-critical flow if $h_u - h_d$ is large and the flow is super-critical immediately on the upstream side.
Fig. 5. Variation of front height with initial downstream depth: (a) parallel–parallel; (b) converging–parallel; (c) converging–diverging; (d) parallel–diverging.

of the wave front. For intermediate values of $h_u - h_d$, there is a second transition from super-critical to subcritical flow on the upstream side of the wave front and this is accompanied by a hydraulic jump. An advantage of shock-capturing methods is that these transitions need not be considered explicitly while performing the numerical computations.

CONCLUSIONS

In the model presented in this study, the two-dimensional unsteady flow equations in a simple transformed coordinate system are solved by the MacCormack finite-difference scheme to simulate the dam-break flow in channel transitions. The numerical results obtained using the present model compare well with the experimental data obtained by Townson and Al-Salihi. The present numerical results also compare well with the previous numerical results of Townson and Al-Salihi based on a characteristic method. The present model is more versatile than the model presented by Townson and Al-Salihi because it is based on a simpler shock capturing method and is applicable to even transitions with curved boundaries. In this sense, the present model is an alternative to a similar model presented by Bellos et al. based on a finite-volume approach.

REFERENCES